BRAINAE SCHOOL OF BUSINESS, SCIENCES, AND TECHNOLOGY

http://www.brainae.org

Email: info@brainae.org +250 788 756 089

SCHOOL: SCIENCE AND TECHNLOGY

MODULE: MATHEMATICS EDUCATION AND HISTORY OF MATHEMATICS

BY: Mr. ISHIMWE Fabrice

Notes on mathematics education and history of mathematics

ISHIMWE Fabrice

May 2022

Table of contents

Lecturer Support Material

This *Lecturer Support Material* has been developed to assist lecturers in the teaching and assessing of *Unit 2: Teaching and Learning Mathematics*.

The material consists of:

- An introduction to the unit, which includes information on the overall rational for the unit as well as recommended teaching approaches and suggestions on how the unit can be integrated into practicum activities. Background readings to support the teaching of the material can also be found in this section
- Module outlines, setting out a suggested sequence of learning activities and identifying topics which could be taught within each module. Ideas for assessment activities are also provided
- A unit Glossary

Suggestions have been made about what content should be covered within the unit as well as recommending an approach to teaching the material. It is envisaged that by working through the suggested sequence of leaning activities lecturers will be modelling good practices for teaching mathematics that students can then apply in their own teaching.

Student Support Material has also been developed for the unit *'Teaching and Learning Mathematics'* to accompany this *Lecturer Support Material*. The *Lecturer Support Material* should be read in conjunction with *the Student Support Material* as the *Lecturer Support Material* makes reference to activities and material contained only within the *Student Support Material*.

It is not expected that the students will work only from the ideas and suggestions contained within the Student Support Material. Additional ideas and activities are set out in the Lecturer Support Material to compliment the student material. The lecturer will need to make decisions about how to present the material and make decisions about what activities will be covered during lecture periods and what students will be required to do during their study time.

When using this material it is recommended that lecturers:

- Read through the whole unit prior to planning the course overview
- Select the modules and topics to be cover in the time available
- Plan the activities that will be presented to the students
- Select the material from the Student Support Material that will be used to support the teaching of the unit
- Develop the assessment tasks for the unit.

It is important to remember that this material is support material only. While lecturers are encouraged to try out the suggested activities within the material, it is hoped that people will also include their own ideas. This material, along with the *Student Support Material* should be seen as a living, working document which can be reviewed and changed to suit the curriculum needs of the Primary Teachers Colleges and new ideas and trends in the teaching of mathematics.

Unit 2: Teaching and Learning Mathematics

The Mathematics course seeks to develop beginning teachers who are:

- Confident in their ability to teach mathematics across the Primary School, are familiar with the Primary School Mathematics Syllabus and have a strong understanding of the mathematical concepts covered within it
- Aware of the factors which impact on the teaching and learning of mathematics
- Resourceful, creative, life long learners
- Inclusive of all people, regardless of gender, social, cultural and language background

Rationale

It is important that teachers develop an understanding of what is meant by mathematics so that informed decisions can be made about what will be taught in the classroom and how it will be delivered. Our philosophy of mathematics will influence these decisions and it is essential that future teachers spend some time considering these issues.

An understanding of how children learn mathematics is essential for all teachers. This should include an awareness of the important role language and culture play in the development of mathematical understandings. The use of concrete materials when introducing new concepts and the importance of establishing connections between the mathematics of the classroom and their application in the community are also factors which teachers need to develop an understanding of.

Teachers need to be familiar with a range of strategies for the teaching of mathematics and be able to cater for all children, including those with special needs. Teachers require the ability to develop appropriate assessment strategies, which recognise a range of learning styles, relate to the full range of curriculum objectives and will provide information that can lead to an improved curriculum.

Many students come to the college with negative attitudes and feelings about their ability to succeed in mathematics. This in turn affects their attitude to mathematics and their approach to the teaching of mathematics. Developing enthusiastic, confident teachers with a positive attitude towards mathematics is essential.

Teachers' beliefs about the learning process influences what occurs in the classroom. For this reason a thoughtful study and understanding of how mathematics is learnt, and therefore how it should be taught, needs to be a high priority for all future primary teachers.

Aims

This unit aims to produce beginning teachers who are:

- Confident and competent to teach mathematics in primary schools (Grades 3 to 8) and familiar with a range of different teaching strategies
- Aware of individual differences and able to cater for all children, including girls as well as boys, and children with special needs
- Familiar with the primary school mathematics curriculum documents and able to use these materials to plan mathematical activities
- Aware of issues which impact on the teaching and learning of mathematics
- Able to use a range of concrete materials to support children in the development of their mathematical ideas
- Able to assess children's learning
- Reflective and able to evaluate their own practice in order to improve their teaching.

Objectives

As a result of studying this unit students will be:

- Competent beginning teachers of mathematics, familiar with a variety of strategies that support the development of children's mathematical thinking.
- Able to identify and discuss a range of factors that contribute to how children learn mathematics.
- Able to identify factors necessary to establish a learning environment that is conducive to the teaching and learning of mathematics
- Able to devise appropriate mathematics activities that cater for all children, including girls as well as boys, children bridging from the vernacular to English, children who have special needs and children in a multigrade class.
- Familiar with the primary school mathematics curriculum documents.

Unit outline

'Teaching and Learning Mathematics' can be offered as either a 2-credit point or 3credit point unit.

To successfully complete this unit students must complete the following core modules:

Module 2.1 Understanding How Children Learn Mathematics

Module 2.2 Teaching Mathematics

Module 2.3 Syllabus Studies

Each of these modules should take between 6 to 9 hours of lectures to complete. It is also expected that students will spend an equivalent number of non-contact time studying the ideas and concepts raised in this unit. A detailed description of each module in included in this Lecturer Support Material. Extension activities have been suggested for those colleges which have allocated 3-credit points for the teaching of this unit.

Sequencing of modules within the unit

When considering the teaching of this unit it is important that the unit be viewed in its entirety rather than as a number of discrete modules and topics. It is recommended that time be made available at the start of the unit to develop an overall view of the concepts and understandings to be developed throughout the semester. The relationships between modules need to be maintained throughout the semester as ideas are built upon and new concepts developed. At the conclusion of the unit it will be important to spend some time reviewing the work covered in the various modules and considering the overall implications for the mathematics in the primary schools. A unit evaluation should also be carried out.

A suggested sequence for delivery of this unit (2 credit points over 12 weeks) is outlined below.

Teaching and Learning Mathematics

Week	Activity
1	Introduction to the unit, including a discussion of each module and the work to be covered
2 - 4	2.1: Understanding How Children Learn Mathematics

5 - 7	2.2: Teaching Mathematics
8 - 10	2.3: Syllabus Studies
11 - 12	Review of the unit, implications for the teaching of mathematics. Unit Evaluation

When teaching this unit it is recommended that ***Module 2.1: Understanding How Children Learn Mathematics***, be taught first. The ideas and concepts covered in this module are considered essential for the development of understandings in all other modules that make up this unit.

It is recommended that the remaining modules be delivered in the order they are listed, that is:

Module 2.2 Teaching Mathematics Module 2.3

Syllabus Studies.

Teaching approaches

The approach that will be adopted to teach this unit will be a student centred, activity based approach. Lecturers will build upon and respect students' different experiences, and provide a range of purposeful and challenging activities. A supportive learning environment will be established, encouraging students to share ideas and to learn from one another.

While exploring the teaching and learning of mathematics students will engage in a range of mathematical activities. These activities will be selected from across the different mathematics content areas and further develop students' own mathematical thinking.

A range of strategies will be used to deliver this unit including:

- Discussions, small group and whole class, open and structured, between student and teacher and among the students themselves
- Seminar presentations
- Research and investigation activities
- Co-operative group learning
- Demonstrations
- Projects
- Peer teaching • Micro-teaching.

Although the unit consists of a number of different modules, lecturers are encouraged to adopt a holistic approach to their teaching. Connections and relationships between the concepts developed in the various modules need to be established and the understandings developed in early modules built upon throughout the unit.

In presenting activities to students, lecturers will be modelling appropriate strategies that can easily be adapted to the primary school context. Class activities, followed by opportunities for group and individual work, recording mathematical ideas and displaying student work, emphasis on the process rather than the product, establishing a classroom environment conducive to learning, and the use of class discussions will be used to deliver this unit. These approaches need to be made explicit to students and consideration given to their effectiveness in the teaching of mathematics.

Inclusive curriculum

In the delivery of this unit it is expected that every person will be provided with the opportunity to participate in and contribute to activities without fear or favour. Activities should be presented to cater for a range of abilities and should be gender inclusive.

When students are considering the teaching of mathematics in the primary school context, attention should be given to catering for all children, including those with special needs. Activities developed for use in the primary school will need to be gender inclusive and present positive and non-stereotypical representations of people.

Language issues

Language factors contribute significantly to children's mathematical learning and mathematics teachers have an important role to play in assisting students to acquire the specialised language of mathematics. Teachers need to establish the connections between the everyday concepts, the everyday language, and the formal language, skills, and symbols of mathematics. Teachers also need to be aware of the language and cultural diversity of children, and how this will impact on the teaching and learning of mathematics.

When teaching this unit, lecturers will need to raise students' awareness of these issues through providing opportunities for students to explore different cultural perspectives and express their mathematical ideas in a variety of ways e.g. through the use of different languages, the use of symbols, graphs, charts and written texts.

When students are investigating the issues raised in this unit, particular consideration will need to be given to ensure that students develop strategies to supporting children who speak English as a second language, and who are in the process of bridging from the vernacular to English.

Multigrade teaching

In implementing this unit lecturers will need to consider how they can cater for the range of student ability levels. By providing opportunities for group work, presenting activities at a range of levels and allowing students to select from these, strategies suitable for use in a multi-grade setting will be modelled. When presenting this unit and considering strategies for teaching mathematics in the primary school context, thought will be given to strategies for teaching in a multigrade classrooms.

Assessment activities

Assessment is the process of identifying, gathering and interpreting information about student learning. The main purpose of assessment is to improve student learning and the quality of the learning programs. Assessment should be undertaken at the beginning of the unit (diagnostic), during the unit (formative) and at the end of the unit (summative).

Diagnostic assessment is the process of discovering what students already know and can do. **Formative assessment** is the practice of building a cumulative profile of student achievement through informal and systematic observations of students. Establishing clear and specific assessment criteria assists in assessing particular outcomes achieved and in providing feedback to students. **Summative assessment** is the practice of making judgements about student achievement at relevant points in the teaching program such as the conclusion of a topic or module. Formal assessment activities such as projects, assignments, tests, and seminars are generally used to make summative judgements.

A variety of assessment strategies should be used and students should be given opportunities, in a variety of contexts, to demonstrate in an authentic manner, what they know, understand and can do. The assessment strategies used need to be sensitive to diversity that exists amongst students and take in to consideration gender, culture, and language differences.

The content that is being covered, the learning objectives being assessed and the style of teaching and learning being used, will influence the method of assessment adopted. When developing assessment tasks lecturers will need to ensure that:

- The requirements of the task are set out clearly
- The assessment tasks chosen are relevant to the objectives and allow students to demonstrate appropriate outcomes
- Marks or grades reflect the relative importance of each part of the task
- The language used is familiar to students and ideas clearly expressed
- Items are not too difficult or too easy
- It does not contain bias
- A marking scheme is developed and applied consistently.

The number of assessment tasks for each unit will be determined by college policy. Assessment tasks have been developed for each module and lecturers will need to select which assessment tasks best meets their needs, taking into consideration the learning objectives for the entire unit.

Suggested assessment strategies for this unit include:

- Oral presentations e.g. Seminars, tutorials,
- Project work
- Research activities
- Peer and micro teaching
- Self assessment
- Reflective journals
- Report writing – with a focus on inquiry, analysis and reflection
- Exams

A range of sample assessment activities can be found in Annex A.

Practicum suggestions

Whilst studying this unit students should be provided with an opportunity to gain experience in the teaching of mathematics in the primary classroom. The following is a list of suggestions as to how this unit may be incorporated into Practicum (including school experience, demonstration lessons, and block teaching). These ideas would need to be negotiated with the Professional Developments strand.

Demonstration lessons

Students observe teachers presenting mathematical activities to children. During these observations students can keep a record of:

- What the teacher is doing
- What the children are doing
- What language is being used
- How the children and the classroom are organised (group or individual work, learning centres)
- What the children are learning
- What problems the children are experiencing
- What concrete materials and teaching aids are being used
- How the children are being assessed.

Following each observation conduct a class discussion reflecting on the lesson. These discussions could incorporate topics such as:

- The prior understandings children needed to participate in the activity
- Identification of the new learning that took place
- The difficulties children experienced and how these could be overcome
- The strategies adopted by the teacher to develop children's understandings and to support the development of mathematical language
- How you could assess the children's learning?
- How you would follow up this lesson?
- Critical reflection on the effectiveness of the lesson and recommendations

School experience

Involve students in microteaching, working with a small group of children over a number of weeks. Students can:

- Plan, teach and evaluate a sequence of mathematics lessons using a variety of different strategies. Students can try out ideas such as:
    - Teaching in context ○ Using concrete materials
    - Developing children's mathematical language ○ Recording mathematical thinking in different ways
    - Supporting children bridging from the vernacular to English when learning mathematics
- Observe a number of different teachers from across the primary school, teaching mathematics lessons. Students can write a journal reflecting on what they have learnt about teaching mathematics from these observations.

At the end of this period spend time reflecting on the experience, sharing what has been learnt and making recommendations for future teaching.

Block teaching

During block teaching students can:

- Plan a series of mathematics activities
- Teach these activities
- Evaluate their teaching

On the completion of block teaching and when students return to the mathematics class, follow up activities should be planned to:

- Share successful experiences and identify the reasons why these experiences where successful (e.g. Good planning, strong understanding of the content area to be taught, use of concrete materials, concepts taught in context)
- Discuss problems experienced, the reasons why these occurred and possible solutions

- Identify the areas where students require additional support and assistance
- Make recommendations for future teaching experiences

Background readings

The following readings have been selected to provide lecturers with an understanding of the issues to be covered in this unit. These can be found in Annex B.

Ministry of Education (Schools Division) (1989). '*How Do We Learn? And How Do We Learn Mathematics?*' in *The Mathematics Framework P-10,* Victoria, Australia

Burns, Marilyn (April, 1993). '*The 12 Most Important Things you can do to be a Better Maths Teacher'* in *Instructor*

Sullivan, P. and Mousley, J.A, (1996). *'Describing and Evaluating Mathematics Teaching'* in Learning About Teaching

Unit evaluation and reflection

On completion of the unit an evaluation should be carried out. This should involve both staff and students reflecting on the teaching and learning that took place during the unit. The information collated during the evaluation process should inform the review and ongoing development of the unit.

Below is an example of focus questions a lecturer may use to review the unit. A Student evaluation form is also included.

Focus questions for lecturer reflection

To determine the effectiveness of the practices and methodologies employed and the content covered in a unit of work, lecturers need to reflect on their teaching. When reflecting on our teaching the areas we can consider are the:

- Content of the unit
- Methodologies used in delivering the unit
- Assessment activities
- Co-ordination of the unit

To help us reflect on our teaching we can ask ourselves a number of questions about each of these areas.

Content of the unit

- Did the content support the objectives of the unit?
- Were the activities sequenced logically?
- Was the content relevant?
- Did the content help the students to become competent beginning primary school teachers?
- Do you think the students are now more confident to teach this subject in the primary school?
- Was the Student Support Material useful?
- What recommendations can I make?

Methodology

- How did I deliver the content to the students? Were these strategies effective?
- Were the students aware of the strategies I was modelling and could they use these strategies in their own teaching?

Assessment of the unit

- How clear were the assessment tasks?
- How many tasks were given to students? Was this sufficient/too few or too many?
- Did I give students enough time to complete each assessment task?
- Do the students' assessment results display what I expected of the course?
- What to the assessment tasks tell me about the students' learning?
- What are my recommendations?

Unit co-ordination

- How well did I co-ordinate this unit?
- Did I produce any materials for students? Were these appropriate?
- Did I communicate well with the other lecturers who were teaching this unit with me?

After considering each of these questions we can then make recommendations about the future of this unit.

Focus questions for Student reflection

Below is a list of focus questions which could be used to stimulate student discussion.

- What have you learnt from this mathematics unit this semester?
- What have been the highlights/strengths of this unit?
- What problems have you encountered with this unit?
- Has this unit helped prepare you to be a beginning primary school teacher? If so, how? If not, why do you think this?
- What comments can you make about the level of work covered in this unit?
- What recommendations do you have about improving this unit?

Student unit evaluations

Unit: _____ Class: _____ *Instructions:*

Put an 'X' in the appropriate box.

Indicators	Strongly Agree (5)	Agree (4)	Not sure (3)	Disagree (2)	Strongly Disagree (1)
1. The objectives of the unit were clearly outlined.					
2. The unit content was clearly related to the objectives.					
3. The student support material helped my understanding of the unit.					
4. The library was able to provide me with additional references.					
5. The assignments were related to the unit objectives.					
6. The instructions to do the assignments were clear.					
7. The assignments were scheduled to allow enough time for preparation.					
8. I obtained useful feedback on my assignments.					
9. Assignments were returned in time to help me with this unit.					
10. Teaching staff were available for consultation.					
11. There were sufficient opportunities to discuss the unit content in class.					
12. Demonstrations and practical activities were useful to my learning.					
13. I have improved in my ability to talk and write about this unit.					
14. I have improved my knowledge and skills in this unit area.					
15. The overall quality of teaching was good					
16. The physical facilities (rooms, labs, equipment) were adequate for the unit					
17. This unit was challenging and at an appropriate level.					
18. I have developed my co-operative learning skills during this unit.					
19. All students (male & female) were provided with an equal opportunity to participate in all activities.					
20. I would recommend this unit to other students.					

Student Unit Evaluation – Notes

The Student Unit Evaluation seeks to determine how students perceive the quality of a Unit though various indicators, objectives, texts, facilities, assignments, and teaching. It is important to note that there is a difference between **Unit quality** and **students' perceptions of Unit quality**. What is being determined here are only students' perceptions. Feedback from students is only one pointer which when linked to other forms of review such as lecturer peer review and self-assessment of a Unit, can provide the basis for improving student learning in a Unit.

From a completed Unit evaluation it is possible to compare the different indicators by calculating a **Mean** score for each of them. Each Indicator Mean score is calculated by multiplying the number of students responding for each preference, by the preference value. The preference values are 5 for Strongly Agree; 4 for Agree; 3 for Not Sure; 2 for Disagree; and 1 for Strongly Disagree.

Example of the calculation of Indicator Mean scores

Number of students: 22

The objectives of the unit were clearly outlined

Indicators	**Strongly Agree (5)**	**Agree (4)**	**Not sure (3)**	**Disagree (2)**	**Strongly Disagree (1)**	**Total**	**Mean**
Student responses	4 (4x5=20)	9 (9x4=36)	3 (3x3=9)	5 (5x2=10)	1 (1x1=1)	76	76/22 = **3.45**

The unit content was clearly related to the objectives

Indicators	**Strongly Agree (5)**	**Agree (4)**	**Not sure (3)**	**Disagree (2)**	**Strongly Disagree (1)**	**Total**	**Mean**
Student responses	2 (2x5=10)	4 (4x4=16)	3 (3x3=9)	8 (8x2=16)	5 (5x1=5)	56	56/22 = **2.54**

Making sense of the results

You can see from these examples what students' general perceptions are. While they thought that the objectives of the Unit were clear, they did not think the content of the Unit was well related to its objectives. A lecturer may have to decide whether stronger links between objectives and content are necessary, whether the objectives should be redefined, or whether students should be made more aware of the links that do exist between objectives and content. To make an informed decision about the Unit, a lecturer would probably need to compare this information with the rest of the evaluation, take note of students' comments at the end of the evaluation, do a selfassessment of the Unit and go through the Unit with other lecturers seeking their opinions (Peer Review).

As a guide, Units probably need fine-tuning where a particular Indicator Mean score is much lower than those of other Indicator Mean scores in that Unit; or any Indicator Mean score is below 3.0. It is possible to calculate an overall Unit Mean by adding all the Indicator Mean scores and dividing by the number of Indicators (20). The overall Unit Mean score can be compared with other Unit

Mean scores in the Strand or across Strands to give a picture of which Units students perceive as being of higher or lower quality.

Resources

The following resources are recommended to support the teaching of this unit.

- Student Support Material for the unit 'Teaching and Leaning Mathematics'
- Butchers paper
- Marker pens
- Department of Education Papua New Guinea Mathematics Syllabus Documents for Lower and Upper Primary
- Mathematics Teacher Resource Books and Pupil Books for Grade 3 to 8
- A range of concrete materials such as blocks, shells, bundles of sticks, rulers, scales
- Access to a primary school classroom

References

Booker, George; Bond, Denise; Briggs, Jack; and Davey, Geoff (1997*). Teaching primary mathematics*, Longman

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*,

Department of Education Papua New Guinea (draft 1999*). Upper Primary Mathematics Syllabus Grade 6-8*

Herrington, Tony; Sparrow, Len; Herrington, Jan and Oliver, Ron (1997) *Investigating assessment strategies in mathematics classrooms*, Edith Cowan University, Perth, Western Australia

Herrington, Tony; Sparrow, Len; Herrington, Jan and Oliver, Ron (1997). *Investigating teaching Strategies in mathematics classrooms*, Edith Cowan University, Perth, Western Australia

Marr, Beth and Helme, Sue (1991). *Breaking the Maths Barrier*, Department of Employment Education and Training, Canberra, ACT, Australia

Ministry of Education (Schools Division) Victoria (1988). *The Mathematics Framework P-10*, Department of Education Victoria, Australia

National Council of Teachers of Mathematics (1994). *Window of Opportunity, Mathematics for students with special needs*, National Council of Teachers of Mathematics Reston, Virginia

Northern Territory Department of Education (1993). Maths in Context, primary units of work, Northern Territory Department of Education, Darwin, NT, Australia

Sullivan, P and Mousley, J. A. (1996). *Learning about Teaching.* Australian Association of Mathematics Teachers, Adelaide, Australia

Annex A – Sample Assessment Tasks

Assignment One: Peer teaching

Due date:

This assignment will require you to plan, teach and evaluate a mathematics lesson. You will be allocated a time to teach your lesson to your peers. This assignment will be worth 30% of the overall marks for the unit. You will be required to present this assignment in three (3) parts.

- Part 1: Lesson Plan, which will be worth 10%
- Part 2: Peer Teaching, which will be worth 10%
- Part 3: Lesson Evaluation, which will be worth 10%

Part One. Lesson plan 10%

Your lesson should be planned for ….. minutes. When writing your lesson plan you will need to include:

- The grade level the lesson has been planned for
- The objective of the lesson
- Resources and materials required to teach the lesson
- Lesson introduction (which should relate to the main objective of the lesson)
- Main body of the lesson (which should include the activities you and the children will do, questions you will ask, examples you will use)
- Lesson conclusion (how you plan to finish your lesson and check that your objectives have been met)

Your lesson plan will need to be presented to the lecturer prior to your teaching.

Part Two: Peer teaching. 10%

Your lesson should go for …. minutes. During your lesson there should be evidence of the following:

- Clear explanation of the activity
- Teaching mathematics in context
- Use of concrete materials
- Opportunity for children to recording their mathematical ideas
- Opportunity for children to discuss their mathematical thinking

Part Three: Lesson evaluation 10%

You are required to complete a journal reflecting on your teaching. In your journal you will need to reflect on:

- Your lesson objective. Was this met and what evidence do you have to support this?
- Successful aspects of your lesson. What went well and why?
- Areas for improvement. What didn't go well and why?

- Changes. What changes would you make if you taught this lesson again?
- Sequencing of the lessons. How would you follow up this lesson?

If you have any questions about this assignment please see your lecturer.

Assignment Two: Developing a mathematics program

Due Date:

This assignment will require you to develop a mathematics program and will be worth 30% of your overall marks for this unit.

For this assignment you will need to choose a particular grade level from the primary school (Grade 3 to 8) and develop a term's program. The program can be for any term e.g. Term 1, 2, 3 or 4.

In developing your detailed program you need to specify:

- The grade level
- The term the program is for
- The units to be covered
- The topics to be covered in these units
- The objectives to be covered for each topic
- The activities you will teach to meet these objectives. When planning your activities you should show evidence of:
    - Teaching maths in context
    - Using concrete materials
    - Allowing opportunities for children to record their mathematical ideas
    - Allowing opportunities for children to discuss their mathematical thinking
- The references and materials you will need
- The assessment strategies you will use

If you have any questions about this assignment please see your lecturer.

Sample Examination

MATHEMATICS / SCIENCE STRAND
Mathematics Exam

Name: _____ Class: ___ ___ Date: _____

Time allowed: 2 hours

Instructions

For this exam you will be required to complete an essay, answering one of the questions written below.
The written essay needs to be at least two pages in length and cover all points stated in the question. Please write your answers on the pages provided and ensure that you have your name written clearly on each page.

CHOOSE ONE OF THE FOLLOWING QUESTIONS

Question A

Co-operative Learning

Co-operative learning is a strategy used in the teaching of mathematics. Discuss your understanding of co-operative learning and the skills you need to be able to work successfully as part of a co-operative group. What do you see as the advantages and disadvantages of co-operative learning as a strategy for the teaching of mathematics?

OR

Question B

Teaching Mathematics

When planning mathematics activities aimed at developing children's mathematical understandings there are a number of issues that a teacher needs to take into consideration. For example:

- children's ability level
- curriculum guidelines
- teaching math's in context
- use of concrete materials
- use of mathematical terminologies
- how to record your mathematical thinking
- create opportunities for children to share their ideas

Choose at least four of these issues and explain why you consider these to be important when developing children's mathematical understandings.

Annex B: Background Readings

The following readings have been included to provide lecturers with some of the necessary background information required to teach this unit.

'How Do We Learn? And How Do We Learn Mathematics?' (1988). in The mathematics Framework P-10, Chapter 3. Ministry of Education (Schools Division) Victoria.

'The 12 Most Important Things you can do to be a better Maths Teacher' by Marilyn Burns in Instructor, April 1993.

'Describing and Evaluating Mathematics Teaching' by P. Sullivan and J. A. Mousley *in Learning about Teaching (1996).*

How Do We Learn? And How Do We Learn Mathematics?

from "The mathematics framework P-10", Chapter 3. Ministry of Education (1988) (Schools Division) Victoria

Language teachers have learned that if the messages in spoken and written language convey little or no sense to the child then the desired learning simply doesn't occur. Earlier approaches which deferred meaning until the child could "read the words" or “learn the letters" are now obsolete. We cannot, for example, ask children to read meaningless sentences like "Run fast, John, run, run, run" and expect them to engage in anything more than the grossest form of rote learning. Meaning is carried by words and is not separate from them.

In mathematics, however, we frequently move students beyond the boundaries of common sense and so learning fails to be spontaneous, natural and meaningful to the learner. For example, messages couched in mathematical symbolism, such as $2g + 6$, frequently fail to communicate any meaning at all and, not surprisingly, no effective learning takes place. If $2g + 6$ is presented without a context of any sort, it will fail to convey meaning, and so, like the child with the graded reader, the process becomes a game where the only available strategy is memory and short term memory at that. The statement is forgotten in a flash because there has been no "real-life" or experiential hooks to mentally attach it to.

Primary teachers, who teach both language and maths, are now transferring the understandings gained from language to their mathematics teaching. They understand that the student doesn't become a different type of learner simply because he or she is learning mathematics.

In post-primary schools, where the separation of the disciplines is well established, mathematics teachers do not know about the advances in language learning that may easily be generalised to their own discipline. Furthermore, they do not recognise any *need* to know about a field which seems so clearly outside the boundaries of their own discipline, mathematics.

Concept maps - a model of how we learn

The theoretical advances made in language learning can serve to inform and improve all learning which takes place in schools and, consequently, are of immediate concern to all teachers, particularly mathematics teachers who have a strong, professional interest in language, specifically mathematical language.

In recent years there has been much research into the processes involved in learning, placing a particular emphasis on what is going on in the learner's mind. The learner attempts to relate new knowledge to what is already known. One model developed to represent this process is that of the "concept map". These maps show linkages, or potential linkages between fragmentary bits of knowledge. As the linkages are made, the concept takes on greater meaning.

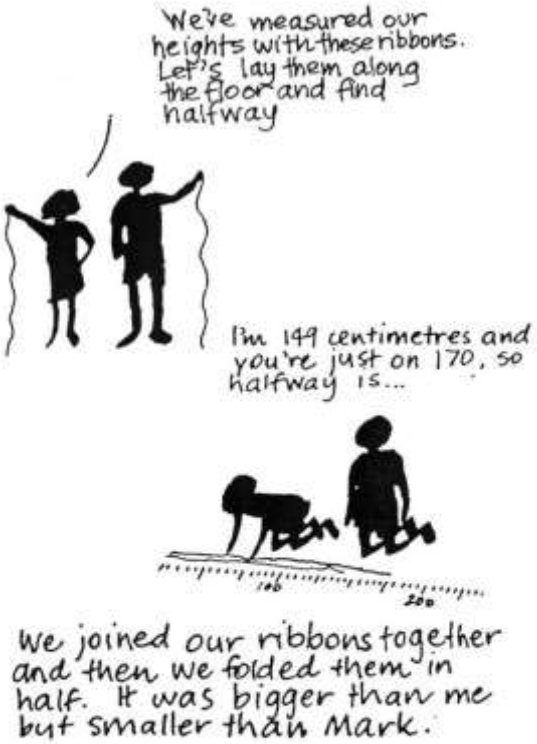
Let us look at this model in the context of the concept of "average" and, in particular, "the mean".

To develop the concept, the notion of "average" must be expressed in ordinary language - language to which the learner can relate.

For example, to find the average height of children in your classroom, you could:

- Line them up, shortest to tallest, and find the middle student.

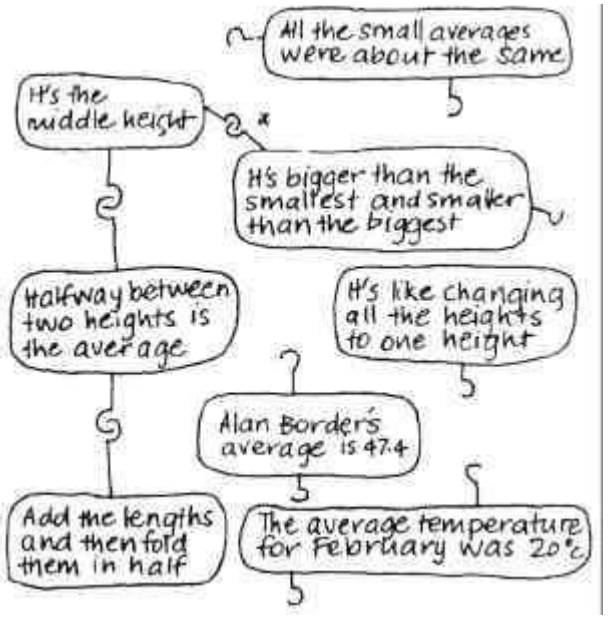
- Pair the shortest with the tallest, next shortest with next tallest, and find the average of each pair - the halfway height. ("What's a way of calculating this?")
- Compare all the paired averages and see how close they are to each other.



With activities like these, students start to "talk-up" the concept in ordinary language, both oral and written. "The average is the number in the middle"; "all the other numbers clump around it"; "There are numbers as far below the average as above it".

It's not exact but that's the point.
Concept development *is* development - a gradual process of improving understanding. And the key to that process of concept development? Language! But, importantly, the ordinary language, written and oral, of ordinary speech. This provides the hooks which link the different parts of the concept map.

Linkings make up a concept



Note that some of the partial concepts from this activity are linked together. Some of the students' real-world experiences have linked to the partial concepts.

Some partial concepts are hooked together; others are not; but all of these partial concepts make some sense on their own.

It is the interaction of the experience them up (measuring children's heights, lining in order, pairing them off) with language which makes the concept of average meaningful, even though the concept at this particular stage may be a very incomplete one.

Through exploratory talk, children use their own words to describe what they are doing, and as they talk so they grasp the concept more fully.

When we understand concept maps we begin to get an inkling of why some student learning doesn't (in fact, can't) take place, in spite of good teaching. There's nothing in the new learning that can hook into the existing concept map. Learning needs to build on present understandings.

Learning is active, not passive

The traditional view of learning has centred on the notion of "transference", the direct transference of knowledge from teacher to student, which to a degree persists even today. What is surprising is that it does so in spite of overwhelming evidence that effective learning does not proceed in this passive way. And if this is true for education in general, then it is doubly true for mathematics in particular.

We can attribute to Piaget and Dienes, among others, the present-day emphasis on activity centred learning in mathematics. They correctly judged that learning is an active, searching process and that a learning environment which is structured for student activity will reflect and support the students' natural learning process.

Learners construct their own understanding of the world; their constructions are personal and are necessarily linked to their own experiences. When the mind experiences something new, older perceptions and concepts are pushed around and shaken up so that the new information can be fitted in with what is already known. Modern approaches to learning emphasise the active, seeking and inventive role of learners. They make sense of their world by actively constructing and reflecting on their constructions, to solve problems and to learn. Learning, according to this view, is a natural process, a process of making sense of things, and children are naturally good at it.

Build on what students know, not what we know

Build on what students know, not what we know. And as we do, we will find that children can and will make sense of mathematics. It won't continue to be that subject where things don't make sense.

Mathematics teaching has a tradition of building on what we as teachers know, and a corresponding tendency to discount what students might know. It may be that we think they know nothing about what we have in mind, until we teach it. Through discussion and listening to our students we may find a way to introduce a topic or embellish it.

Quite often the clue to how we go about teaching a particular topic resides within the student and not with us.

Our perspective is mathematics, the discipline; students' perspectives arise from their real-world experiences and interests.

Mathematics, like all subjects taught in schools, can be culturally biased. Children's knowledge of their world will be influenced by their social and cultural environment. An understanding of these factors is needed to utilise the richness of students' experience. For example, because of family or gender-role expectations, a child from a non-English-speaking background may have a very different outlook from an Australian child on ways of life or the individual's role in society; Aboriginal children will have different educational needs and make a different contribution from their non-Aboriginal classmates. Ministry guides for teachers of Aboriginal children (1984 and 1986) and *Number? Maths* (Thomas 1986) provide illustrations of, and give suggestions for addressing, these issues.

Starting from Aden's real world view of maths: a case study

Aden, a Year 5 student at a Brunswick school, could make no sense of the standard method of decomposition subtraction. He had the language wrong and had no idea even of what a reasonable answer might be to this subtraction:

500

- 45

———

———

He knew his answer of 365 was wrong but was unable to explain why. Later, asked how much change he would get from $5 if he bought a 45-cent chocolate bar, he said: "Four dollars, fifty-five cents. 1 took one dollar away from the five dollars. Then 1 took 45 cents from the dollar which left me 55 cents. Then I brought the $4 back. That gave me $4.55" (video: *Real Maths, School Maths).*

Notice that Aden's language description closely mirrors the traditional method of decomposition subtraction, but in this case he has made things easier for himself by not decomposing the one dollar into tens and units. Instead he has decomposed the dollar into 100 cents and then performed the subtraction, 100-45, mentally.

Aden's approach recognises that with money it is easier to decompose the 100s (the dollar) into units rather than tens and units.

He has, in fact, a good understanding of subtraction which we might have missed if we had only presented him with pen-and-paper methods in abstract contexts.

Students need to talk and write about mathematics

It may be that in our haste to move students along to the most refined level of mathematical symbolism we leave out the critical step of providing opportunity for conversation, discussion, challenging one's own guesses and having them tested by other members of the group ... interactive talk. A lot of cognitive reorganisation goes on during these opportunities for talk.

In like manner, writing in ordinary language about a mathematical experience such as finding a class average for height or exploring a mathematical process as Aden did, can be a powerful way of exploring a concept or process. Writing, unlike talk (which is transitory and lost in the flow of time), is a more formal and permanent record of what has been done and what understandings have been obtained. Writing is more open to question, more available for reconsideration, and more likely to promote awareness of one's own thinking.

Both talking and writing in ordinary language about mathematical concepts help to increase this awareness and so help to move the learner from thinking which is embedded in the context of real life, to the thinking that Margaret (1984) describes as "disembedded thought' - thought which

abstracts the essence of a concept from its contextual background. And this is what mathematics is about.

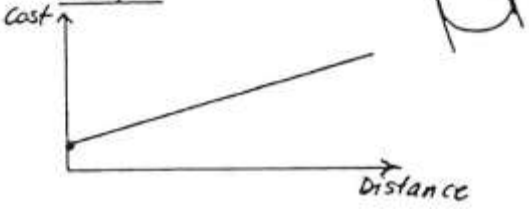
Mathematical language finally moves to a form of symbolism entirely divorced from any context. The route to this most advanced form of mathematical language is ordinary language.

We need to consider those children whose mother tongue is other than English. Problems that may be encountered and techniques for resolving these are discussed in *The LOTE Framework* – "Types of Programs".

A model for mathematics learning which includes ordinary language

Mathematics begins with the real world. It motivates our search for pattern, explanation and prediction. We each hold an internal view of real world events (such as adding twenty cents and thirty cents) and this view is based on our perceptions and understandings, our "concept map" - in this case of money: how it is added, spent and exchanged. Mathematics learning can be characterised by the following six stages.

Stage 1 THE REAL-WORLD EVENT Starting with a real-world or constructed event establishes a common point of reference and a context from which students can draw understandings.	**Topic: Year 9 Linear Equations** Investigating the fare structure of the public transport system, the 'MET'.
Stage 2 COGNITIVE REPRESENTATION Students' cognitive representation of real world events depends on their own experiences. What are these experiences? What do students know?	Start with a story or discussion. How does the MET calculate the cost of a suburban fare? 

Stage 3 ORDINARY LANGUAGE Students talk to develop, explore and test their understandings of concepts. Students' talk is a "window" which enables us to "see" their internal representations of an external event. Ordinary language, written and oral, serves both learning and teacher purposes. It is the critical underpinning for more directed and precise mathematical language in Stage 4.	What are the costs? There are fixed costs and costs related to how far you travel. 
Stage 4 MATHEMATICAL LANGUAGE A mixture of ordinary language and mathematics language operates as a "halfway house" and thinking becomes progressively more abstract. By "mathematical language" we mean the formal language of mathematics. Once students have understood and internalised concepts, they can be encouraged to use the formal and precise language of mathematics. Like informal language, mathematical language can be written or oral.	Cost = $1.25 plus 7 cents per kilometre Cost = $1.25 + 7c for 1 km. Cost = $1.25 = 7c × 2 for 2 km Equation C = 1.25 + 0.07d where C = cost and d = distance in kilometres Table d (km) \| Cost ($) 0 | 1.25 1 | 1.32 2 | : 3 | : Graph 
Stage 5 MATHEMATICAL SYMBOLISM The final, most refined form of mathematical language is abstract, brief and precise. It can be understood if it is supported by both mathematical language and ordinary language.	

Stage 6 TRANSLATION BACK INTO REAL-WORLD TERMS The results of mathematical operations with symbols are translated back into real-world terms and made sense of. How does our mathematical solution relate to our initial problem?	

Stages 3 and 6 are the essential stages that have been left out of the concept development. They are the critical stages which give meaning to what is done in the name of mathematics, and for most students that is precisely what their mathematical learnings have lacked.

There are two key elements in using ordinary language in mathematics classrooms:

- Students must talk in order to assist, clarify and test their developing concepts. (See section on Group-work and Problem-solving.)
- Teachers must listen to what students have to say. That is, we must value their present knowledge, no matter how raw it is. Listening to students will suggest a wealth of "start strategies".

You'll notice that this is a complete reversal of what typically takes place in mathematics classrooms. So - how do we go about it?

Using ordinary language: some start strategies

Here are some ordinary-language "start-strategies" which illustrate how the mathematical concept can be linked to students' experience and the associated use of language. The examples draw from both computation and "manipulative" algebra. Both these areas present students with considerable difficulty and, not surprisingly, both have heavily emphasised "manipulation", either of digits or symbols. Both have lacked a reality base, and their formal symbolic meaning escapes many students.

Year 6. Division with large numbers

Instead of teaching meaningless long division with numbers like 1541 ÷ 92, use estimation and the calculator.

Link division to a meaningful context

"Brisbane is 1541 kilometres from Melbourne. I can average 92 kilometres each hour. How many hours would it take me to drive?"

Use approximation, estimation and number patterns

Notice that by linking division to real experiences children can move back and forth from "pure" number to reality, all the time using reality as a check on their reasoning.

Use a calculator if you need more accurate answers

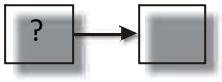
1t's 17.1222. Well that's about 17 hours. That's OK, we said a bit more than 15."

Years 7 and 8: Solving algebraic equations such as 2(2n + 3)-1 = 21 Each of the words in the mathematical phrase "Solving algebraic equations" is meaningless to the average student at this level.

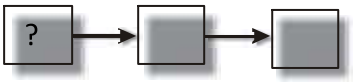
Start with a "magical experience" in ordinary language

Don't choose a real-world event but a constructed event that touches on children's enthusiasm for activities such as puzzles or guessing the number.

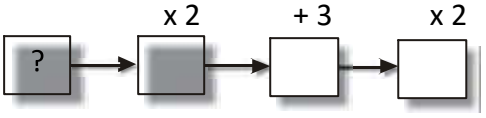
Students write a "mystery number" on the board, known to the class but not to the teacher. x 2



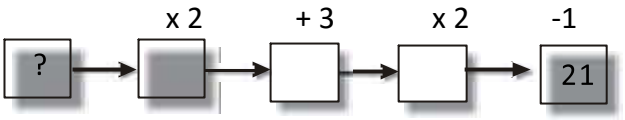
"Now multiply the number by 2." x 2 + 3



"Add 3."



"Now multiply that by 2."



"Finally, take away 1."

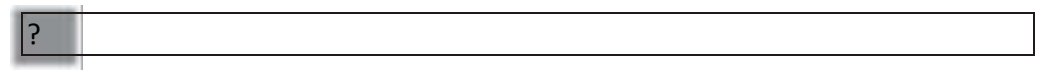
Teac her: "What's the final answer?" C lass: "21!"

Teac her: "The mystery number is4!"
C lass: " How did you know?"

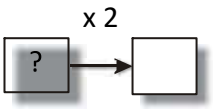
and with that the teacher proceeds to unravel the mystery. The clue? Ordinary language. 1f 21 was in the last box what number do you think must have been in the second last box before 1 was taken away?" "22". And so, teacher and class proceed to "Backtrack" (RIME) their way back to the starting number. If 22 was the answer in the second last box before the number was doubled, what do you think must have been the third last box?

Link ordinary language to the symbols

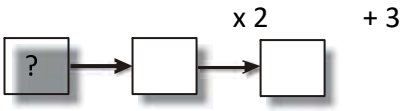
We can build up the equation with ordinary language, and we can do it with symbols.
From the previous example:



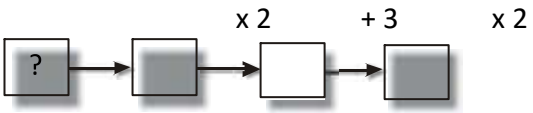
bec omes n



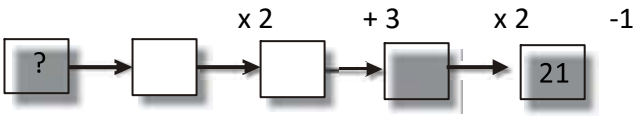
bec omes 2n



bec omes 2n + 3



bec omes 2(2n+ 3)



bec omes 2(2n+ 3)-1

Finally: 2 (2n + 3) -1 = 21

Similarly, we can solve (or unbuild), starting with 2 (2n + 3) -1 = 21:

"If 21 was the answer *after* 1 was subtracted, what was the number *before* it was subtracted?" 22.

In other words, 2 (2n + 3) = 22.

Risk-taking and error-making are natural to problem-solving

Most effective learning involves risk-taking. In order to think and act creatively students need to be in a learning environment which supports risk taking. There are occasions when we should accept a reasonable answer rather than insist on the right one. If the emphasis is always on getting the "right" answer, students will be much less likely to "have a go", consider alternatives, guess, hypothesise and play around with exploratory thinking; in short, to adopt divergent thinking processes.

Effective problem-solvers expect to be wrong on occasions - to make errors. Errors are a valuable source of information and feedback to the learner. Learning that expects to be right all the time proceeds timidly.

Learning that accepts (and expects) error is natural and adventurous, but most importantly, effective. That which is learned makes sense. It sits comfortably with what is already known. It forms part of the learner's experiential knowledge of the world.

Acceptance of approximate responses

In language teaching we have learned to accept that in the process of learning to speak, read and write children make guesses, approximations; sometimes "near enough is good enough". For example, when children are reading we don't interrupt the flow if it is meaningful and making sense. We accept approximations. In maths the evidence suggests we should do likewise.

You need to feel good about yourself to tackle mathematics successfully Following closely on the idea of risk-taking as essential to learning are the notions of self-esteem and success. The notions, of course, are circular: success enhances self-esteem, and high self-esteem encourages risk-taking which contributes to effective learning and further success.

Students need a supportive environment in which they feel free to make mistakes and engage in exploratory thinking as part of the learning process. (The section on "Students Working Together in Groups" outlines a number of ways in which such supportive environments can be fostered in the maths classroom).

The 12 Most Important Things You Can Do to Be a Better Maths Teacher

by Marilyn Burns (in 'Instructor', April 1993)

Not too long ago, teachers saw the main goal of maths instruction as helping children become proficient in paper-and-pencil computation. Today, mathematics instruction is less about teaching basic computation and more about helping students becoming flexible thinkers who are comfortable with all areas of mathematics and able to apply mathematical ideas and skills to a range of problem solving situations.

Making these goals a daily part of maths instruction may require a shift in the way you think about mathematics and your role in teaching it. And so I offer the following suggestions and examples from actual classroom lessons to help you re-think your teaching practices.

1. Set the following expectation for your students: Do only what makes sense to you.

Too often, students see maths as a collection of steps and tricks that they must learn. And this misconception leads to common recurring errors in arithmetic calculations – when subtracting, students will subtract the smaller from the larger rather than borrowing: when dividing, they'll omit a zero and wind up with an answer that is ten times too small: and when combining fractions, they'll incorrectly add both the numerators and the denominators. In all instances students arrive at an answer that makes no sense, and they neither notice nor care.

There is no place for children doing mathematics by rote. Students must be able to explain the purpose of what they're doing, the logic of their procedures, and the reasonableness of their solution.

2. Have your students explain their reasoning in all instances

It is insufficient and short sighted to rely on quick, right answers as indicators of students' mathematical power. During maths lessons, probe children's thinking when they respond. Ask; why do you think that? Why does that make sense? Convince us? Prove it? Does anyone have a different way to think about the problem? Does anyone have another explanation?

When children are asked to explain their thinking, they are forced to organise their ideas. They have the opportunity to develop, cement, and extend their understanding. Teachers are accustomed to asking students to explain their thinking when their response is incorrect. It's important, however, to ask children to explain their reasoning at all times, even when their answer is correct.

3. Encourage children to talk to one another during the maths class.

Communicating is essential for learning. Having students work quietly – and by themselves – limits their opportunities for learning. Interaction maximizes children's opportunities to talk about their ideas, get feedback for their thinking, and hear other points of view. The challenge today is to have students engage in dialogue and work together to solve problems and bring meaning to mathematics ideas. Students can learn from one another, as well as from the teacher.

4. Make writing an integral part of maths learning.

Communication in the maths class should include writing as well as talking. In his book, *Writing to Learn*, William Zinsser states 'Writing is how we think our way into a subject and make it our own.' When children write in the maths class, they have to revisit their thinking and reflect on their ideas. Student writing gives teachers a way to assess how their students are thinking and what they understand.

Writing in the maths class best extends from children's talking. When small group interaction or a whole class discussion precedes a writing assignment, students have a chance to formulate their ideas before they're expected to write. It's also helpful to write prompts on the board for students to get them started. For example "I think the answer is _____. I think this because _____".

5. Embed the maths activities in contexts.

When connected to situations, mathematics comes alive. Contexts give students access to otherwise abstract ideas. They stimulate student interest and provide a purpose for learning mathematics. Contexts can draw on real-life examples, or they can be created from imaginary situations. Many children's books for example offer starting points for classroom mathematics lessons.

6. Use manipulative materials whenever possible.

Manipulative materials provide a concrete context for thinking about mathematics. They give children hands-on experiences for grabbing onto mathematical ideas, turning them around, and viewing them in a different way. Manipulative materials can serve in several ways – to introduce concepts, to pose problems, and to serve as tools to figure out solutions. It's important that manipulatives are not relegated only to young children, but that they are made available to students in every grade.

7. Bring the quality and richness often apparent in student's writing and art into their maths work.

Typically in primary classrooms, children's rich, varied, and creative art and writing fill the walls, while the maths work that is posted commonly consists of arithmetic worksheets or progress charts that tracks students' learning of basic facts. And in language arts and art lessons, children's imaginative ideas are invited and applauded; unique and unexpected results are common. In maths lessons, students often learn and practice specific prescribed methods, consistency among student papers is desired.

Find ways to make maths lessons and assignments as intriguing, rich, and motivating as they are in other areas of the curriculum. Give students the chance to use their creativity when thinking about mathematics. Encourage them to be inventive and trust that they will be.

8. Where possible make calculators available to children.

Calculators are valuable tools for doing laborious mathematical computations and a basic tool few adults do without. As stated in the NCTM *Curriculum and Evaluation Standards for School Mathematics* (page 8): "Contrary to the fear of many, the availability of calculators … has expanded students' capability of performing calculations."

Calculators are not a replacement for student thinking. In any problem situation, a child needs to know which buttons to push, whether the answer displayed makes sense, and what decisions to make with the results. Calculators eliminate the drudge of complex calculations. They also help children solve problems they might not otherwise be able to tackle.

9. Let children push the curriculum rather than having the curriculum push the children.

Choose depth over breadth. David Hawkins has said, 'You don't want to cover a subject, you want to uncover it" (*The Having of Wonderful Ideas* by Eleanor Duckworth, Teachers College Press 1987). There are many pressures on teachers, and the school year passes very quickly. But students' understanding is key and doesn't always happen according to the schedule suggested in the text materials. Just as students should do only what makes sense to them, the same is true for teachers. There is value in staying with a topic children are interested in, pushing more deeply, and taking the time for a side investigation that can extend a lesson in a different direction.

10. Keep an eye out for instructional activities that are accessible to students with different levels of interest and experience.

A wonderful quality of good children's books is that they delight adults as well. Of course, adults appreciate books for different reasons than children do, but enjoyment and learning can occur simultaneously at all levels. The same holds true for mathematics investigations. Search for activities that can engage children who have the least mathematical experience while challenging students with the most experience.

11. Remember that confusion and partial understanding are natural to the learning process.

Do not expect all children to learn everything at the same time, and do not expect all children to get the same message from every lesson. Although teachers want all their students to be successful, they rarely reach every student with any one lesson. Learning should be viewed as a long-range goal, not as a lesson objective. It's important that children do not feel deficient, hopeless, or excluded from learning mathematics. The classroom culture should reinforce the belief that errors are opportunities for learning and should support children taking risks without fear of failure or embarrassment.

12. Take delight in students' thinking.

There is no one way to think about any mathematical problem. Encourage students to think in different ways. After children respond to questions (and of course, have explained their thinking), ask; Does anyone have a different idea? Keep asking until all children who volunteer have offered their ideas. By encouraging participation, you'll not only learn more about individual children's thinking, but you'll also send students the message that there is more than one way to look at any problem or situation.

Describing and evaluating mathematics teaching

By P. Sullivan and J. A. Mousley, in 'Learning About Teaching' (1996), Australian Association of Mathematics Teachers

The application of so-called total quality management principles to education has directed considerable attention to the identification of quality practices and to the evaluation of teaching. The professional standards for teaching mathematics (NCTM, 1989) are acknowledged across all areas of teaching world wide as a watershed document which has defined the debate. An important next stage for our community is to develop a professional language which can be used to describe teaching in order to facilitate evaluation.

This article is an attempt to contribute to the discussion on ways to describe and evaluate aspects of mathematics teaching. It presents a list of components which arose from a survey of teacher educators on ways to describe quality mathematics teaching, and attempts to show how these can inform our understanding of the task of teaching.

Components of quality mathematics teaching

The components of quality lessons listed below were determined from the commonly identified features of teaching presented in response to a survey item. The survey asked 125 teacher educators to imagine a high quality mathematics lesson, "at any year level, where the students are learning, for example, to estimate the mass of various objects, or to add fractions, or to record given information as a graph". The responses were coded and organised with the assistance of a qualitative data analysis package (see Mousley, Sullivan and Waywood in press, for a full description of the method).

The six components distilled from the responses to the survey are not intended to describe quality teaching exhaustively; but it is suggested that most of these features should be present in high quality lessons. They are outlined below:

Building understanding refers to a role which the teacher assumes in order to convey some pre-determined meaning to students. It is a recognition of particular understandings to be developed, and of strategies to achieve these ends by building on existing knowledge, using materials to explain and clarify concepts, choosing appropriate sequences, helping students to make connections, forming relationships, and knowing the meaning of terms. There is a strong inference of teacher decision, teacher direction, teacher explanations and teacher control.

Communicating relates to opportunities for talking, explaining, describing, listening, asking, clarifying, sharing, writing, reporting, and recording. It includes class organisation structures such as co-operative groups, and is characterised by an orientation of teacher and students to accepting communication as a two-way process.

Engaging refers to facilitating student involvement in their own learning. It includes engaging students in mathematical activities, and motivating students to learn. Teachers choose content and materials which are personally relevant for the students or based on real-world situations, and seek to make learning enjoyable.

Problem solving is when students work out for themselves how to perform mathematical tasks in such a way that it is the students' own work and they know that it is. It refers to activities such as risk-taking, challenging, exploring, investigating, thinking, asking, and posing.

Nurturing refers to the establishment of a relationship between the teacher *and pupils.* Teachers do not seek just to teach mathematics but to teach students as well. While no doubt teachers of mathematics endeavour to cater for the range of abilities in their class and to develop rapport with their students, nevertheless this is more a recognition that teaching and learning is a two-way process and that there is something natural in the expert/novice relationship, which includes a nurturing component.

Organising for learning describes comments related to decisions made by the teacher about a specific focus for what would happen in the lesson and a commitment to pursuing that focus and to communicating the focus to the students.

A number of important issues arise from the attempt to clarify the meaning of the words used to describe these components. For instance, *Building understanding* suggests the development of concepts and mathematical ideas as well as developing conceptual links. It perhaps most refers to the Skemp (1976) notion of relational understanding. It requires an orientation on the part of the teacher to plan and teach mathematics in an orderly, coherent, and connected way which supports sound, flexible conceptual development and elaboration of mathematical structures.

It may be useful to register some disclaimers. It does not refer to that approach where mathematics is presented to students as a collection of isolated rules and tricks to be applied on cue. Nor is *building understanding* compatible with the perspective which emphasises the students' creation of their own mathematics. For example, Bauersfeld (1991) defined learning as "... a process of

personal life-forming, a process of an interactive adapting to a culture through active participation ... rather than a transmission of norms, knowledge and objectified items" (p. 273).

Building understanding is different from this and assumes a different view of the nature of mathematics itself. There is an implication of a culturally defined view of mathematics negotiated over centuries and into which we would seek to induct the young. Nor is *building understanding* necessarily evidenced by the use of materials. Materials themselves can become the focus of instruction rather than building understanding. An examination of some elementary school mathematics texts in the 1970s and 1980s confirms this.

It is clear that we need to find ways of observing examples of teaching in order to articulate the practical meaning of the term so that we can identify and discuss the building of understanding when we see it.

Similar comments apply to the component *Communicating,* which is widely acknowledged as an essential component of learning mathematics. The term, *however,* is often used without an adequate definition or explanation which would allow an appreciation of what is actually meant. Discussion within a mathematics class is not evidence of effective teaching. This suggests that the terminology we use to describe this aspect of teaching needs to be more explicit and less open to interpretation by the reader.

Clearly students talking to each other about mathematics is an avenue to other goals. It has limited value in itself. If we found that students learn mathematics best by working by themselves (such as in piano practice) then we would encourage students to do this. We only emphasise discussion, for example, because we know that it helps student to learn.

A lack of clarity is also associated with some phrases used to construct the component *Engaging*. For example, phrases which included words like "active involvement' were frequently cited by respondents to the survey. However, by itself the term "active involvement has no real meaning. Does it presume some physical activity? Is physical activity enough? This is another example of a term which is used loosely and which serves to reinforce the readers' or listeners' predisposition but which does not necessarily convey the meaning intended by the communicator.

Also grouped within the component *Engaging* were frequent references to the use of "real world" examples. Is the use of a real world example merely an avenue to some other goal, is it a goal in itself, or some combination? This perhaps is an issue which could provoke some debate among mathematics teachers, but it is clear that the issue is not resolved by examination of curriculum documents. On one hand, it is only through an understanding of generalisable, principles that students can then apply their mathematics to unfamiliar situations. In this view, the applications merely become illustrations of the mathematics being taught.

Another perspective would see practical applications as the only tangible expression of mathematics and some argue that if aspects of mathematics cannot be taught through practical situations then those aspects should not be taught at all. It would seem useful for us to engage in more dialogue to seek to resolve such competing perspectives.

The component *Problem Solving* has been a major strand in considerations about mathematics teaching and learning for over thirty years (e.g. Polya, 1957). It is not surprising that some variation of the term problem solving was used frequently by respondents to the survey. Nevertheless the term has been used to mean many things in many situations. There was a stage where problem solving meant resolving puzzles about foxes, wheat and canoes. However, as Siernon and Booker (1989) explained, there are some students for whom calculating the number of handshakes in a room is not a problem whereas there are others who do not have a ready strategy for solving 24 -&- 6 and so this is a problem. As with other terms discussed here, use of the term problem solving does not convey to an intended audience any specific meaning.

Linking the components together

While these six components present a summary of the responses to the survey, they are different in both focus and locus of responsibility. It seems that *building, understanding, nurturing and organising for learning* are initiated by the teacher, while *communicating, problem solving,* and *engaging* are done by the students.

It also seems that *building understanding* is somewhat more central to the task than the others. Building understanding was easily the most commonly cited component of quality teaching. Upon reflection, it seems that each of the other components could be considered as vehicles for building mathematical understanding. For instance, organising for learning seems directed at teacher actions which in turn lead to building understanding.

Of course, teacher activity is not divorced from student activity, and there are reflexive relationships between such aspects as engaging and problem solving as well as engaging and building understanding. What is highlighted though is that if we seek to evaluate our teaching we must be clear on the real goals. For example, engaging the students, by itself, may not be enough. The engagement must be in activities which enhance the building of understanding and learning of mathematics. Likewise, teachers should know that communicating is for building understanding, (and indeed for enhancing engagement and problem solving which in turn lead to building understanding), and by itself is not evidence of effective learning. Just because students are talking to each other, or working in co-operative groups, or asking the teacher questions, is not grounds for a positive evaluation. These actions must be directed to some purpose and it is against this purpose that the evaluation must be made. In the same way, using real life examples does not ensure effective learning and certainly by itself may not contribute to building understanding.

Classroom tasks for quality teaching

A further issue which is central to this discussion is the selection of classroom activities for the student. Even if a teacher can adopt the appropriate orientation to teaching mathematics, the choice of the classroom tasks and activities will have a significant impact on the learning outcomes and the view of mathematics which students will, adopt. It is necessary to consider the style of classroom task. (The reverse. is also true: carefully selected tasks will not be sufficient by themselves.)

Christiansen and Walther (1986) suggested that teaching should be based on carefully chosen activities of the constructive, exploratory and problem solving types which allow the teacher to build on the learner's personal activity. They give an example of a task:

How much does it cost to feed a dog?

The teacher might hope that pupils construct a plan, organise the work, collect data, systematise, transform, reorganise, and evaluate. The intended learning is the development of processes. The teacher's role is to differentiate between learning needs according to required levels of support. The hope is that students' actions develop into cognitive strategies and over time become schemata.

Other much quoted tasks of the style which Christiansen and Walther recommended are:

Design a new playground (car park) for your school.

Compare the bounce, of these four different balls.

Lovitt and Clarke, (1989) compiled an outstanding collection of investigations of such realistic situations. Examples of their tasks include:

How many different houses made from four cubes can you design? How many people can stand on your classroom? and

How long would it take for $1,000,000 worth of cars to drive past a point on your road?

Good questions

One approach which is based on the use of open-ended tasks was described by Sullivan and Clarke (1988). They used the term "good questions" to describe a style of open-ended tasks which are also content specific. One example, of a good question is:

My vegetable garden is shaped like a rectangle. The perimeter of the, garden is 30 metres. What might be the area of my garden?

This question is different from conventional perimeter and area questions in two major ways. First, it requires a higher level of thinking and engagement than do conventional questions. Traditionally mathematical questions have required students to repeat a procedure or recall an algorithm. The sample question engages students in constructive thinking by requiring them to contrast the related concepts of perimeter and area and to think about relationships for themselves. Another advantage of the question over conventional items is that the need for thinking by individual students is made clear to the student. The students cannot rely on remembering a rule or simply manipulating formulae, they must think about the concepts, their *meaning* and the links between them.

Second, the question has more than one possible appropriate answer. Some students might give just one response, others might produce many appropriate answers, and there may be some who will make general statements. The openness of good questions offers significant benefits to classroom teachers because of their potential for students at different stages of development to respond at their own level.

Among other advantages for classroom teachers is that good questions are suited to group learning, and they focus the attention of students on to aspects of mathematics such as generalising, and identifying patterns and relationships. Good questions allow students to be creative, to work with others in responding to set tasks, and to recognise that many problems have multiple solutions. An important feature of the questions is that learning occurs as an outcome of the students' explorations and thinking, not necessarily as a result of listening to the teacher.

Other examples of open-ended tasks which can be called good questions are:

A number has been rounded off to 5.8. What might the number be? Draw some triangles with an area of 6 sq. cm. Find two objects with the same mass but different volume. Describe a box with a surface area of 94 sq. cm.

The role of the teacher in lessons based on potentially rich mathematics questions is by no means obvious. However, focusing on one or more components of quality teaching can act to give direction. If, for instance, building understanding is the focus, then this does help the teacher to identify an appropriate lesson structure and it also gives indications of when to intervene and when to hold back. A general guide for using good questions as a basis for the teaching and learning of mathematics may be that teacher explanations should come after students have had the opportunity to work on the class tasks.

Focusing on components of quality teaching

Recent developments in technology provide us with additional tools to allow us to study and learn about teaching. For instance, based on a belief that the presentation of examples of different

components of teaching can be a stimulus to discussion and reflection, as well as to the development of more common understandings of the terms that we use to describe teaching, the authors have prepared a resource where examples of teaching stored on CD-ROM are selected through a series of menus. The resource also contains lesson documents, transcripts, graphics and other representations of the classroom interaction. The use of CD-ROM allows very fast accessing of different sections of the video, and immediate replay of incidents. It is possible, for example, to view all of the incidents of "pupil-to-pupil communication" one after the other, or to examine all of the interactions of any one child with the teacher merely by selecting that option. This allows users to clarify with each other what they mean by the various terms they use when applying the terms to specific examples.

Summary

The six components of quality mathematics teaching listed above were distilled from responses of 125 experienced mathematics educators to an open survey item which sought views on features of quality lessons.

There were two points which arose from the responses. First, the professional community of mathematics teachers does not have concise technical language to describe the activities which constitute our professional work. There is a need to identify significant components of the teaching task, and then to seek to describe these components clearly. This could then form the basis for a more rigorous examination of the actual foci of the mathematics education community's planning, actions and reflection - which leads to the second point, that our evaluation and reflection must include consideration of our real goals. It is by no means clear that there is any consensus on our primary task. Nevertheless we should seek to examine critically each of the components of teaching and to differentiate between means and ends.

A critical aspect of the implementation of an approach to quality mathematics teaching is the choice of the task. This issue also would benefit from more intense discussion among the community of mathematics teachers.

References

Bauersfeld, H. (1991). Integrating theories for mathematics education. In R.G. Underhill (Ed.), *Proceedings of the thirteenth annual meeting of the North American chapter for the Psychology of Mathematics Education (pp.* 246-269). Blacksburg, VA: PME.

Christiansen, B. and Walther, G. (1986). Task and activity. In D. Christiansen, A.C. Howson, & M. Otte (Eds.), *Perspectives on mathematics education (pp. 243-307).* Amsterdam: Reidel.

Lovitt, C. and Clarke, D. (1989). *Mathematics curriculum and teaching program activity bank,* Volume 1 & 2. Canberra: Curriculum Development Centre.

Mousley, J. A., Sullivan, P., and Waywood, A. (forthcoming). Using a computer in synthesis of qualitative data. In A. Teppo (Ed.), *Qualitative research in mathematics education: theoretical and methodological perspectives.*

National Council of Teachers of Mathematics (1989). *Professional standards for teaching mathematics.* Reston, VA: NCTM.

Polya, *G.* (1957). *How to solve it. New York:* Doubleday.

Siernon, D. and Booker, G. (1989). Teaching and learning for, about and though problem solving. Vinculum 27, June, pp. 4-5.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77, 20-26.*

Sullivan, P. and Clarke, D. (1988). Asking better questions. *South East Asian Journal of Science and Mathematics Education Xl,* June, pp. 14-18.

Module 2.1 – Understanding How Children Learn Mathematics

Module 2.1 Understanding How Children Learn Mathematics is a core module in the unit ***'Teaching and Learning Mathematics'***. It is recommended that this be the first module taught within the unit, as the topics covered here will inform the understandings to be developed in latter modules.

During this module students will extend their understanding of what is meant by the term 'mathematics' and explore the relationship between culture and mathematics. The importance of language in the teaching of mathematics will be examined. Active participation, relating mathematics to real life situations and the use of concrete materials to support the development of mathematical concepts are issues which students will study. The understandings developed in this module will enable students to make informed decisions when planning and teaching mathematical activities.

Objectives

By the end of this module students will be able to:

- Develop a definition of what is meant by the term 'mathematics'
- Explain how culture influences mathematical thinking
- Justify the use of concrete materials in the teaching of mathematics
- Discuss the relationship between language and mathematics
- Identify strategies for actively involving children in learning mathematics

Concepts and skills to be developed

- Relationship between language and mathematics
- Teaching mathematics in context
- Active participation
- Oral presentation (debate, seminar)
- Research skills

Topics

- What is this thing called 'mathematics'
- Mathematics and culture
- Language and mathematics
- Concrete materials
- Active involvement
- Teaching in context

Suggested teaching strategies

- Class discussion (whole class, small group)
- Debate
- Material production
- Small group work
- Independent study

Suggested assessment tasks

- Debate the role of ethnomathematics in the teaching of mathematics
- Journal: Discuss the role of language in the teaching of mathematics
- Develop a mathematics lesson which demonstrates children's active participation • Develop a teaching aid suitable for use during a mathematics lesson

Resources

- Student Support Material for the unit 'Teaching and Leaning Mathematics'
- Butchers paper
- Marker pens
- Department of Education Papua New Guinea Mathematics Syllabus Documents for Lower and Upper Primary
- Mathematics Teacher Resource Books and Pupil Books for Grade 3 to 8
- A range of concrete materials such as blocks, shells, bundles of sticks, rulers, scales
- Access to a primary school classroom

References

Booker, George; Bond, Denise; Briggs, Jack; and Davey, Geoff (1997). *Teaching primary mathematics,* Longman

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5,*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Marr, B. and Helme, S. (1991). 'Language and Maths' in *Breaking the Maths Barrier,* Department of Employment and Training, Canberra, Australia

Mousley, J. and Sullivan, P (1996). *Learning about Teaching, an interactive tutorial program to facilitate the study of teaching.* AAMT, Deakin University and The Australian Catholic University

A suggested sequence of learning activities

Introduction – What is this thing called 'mathematics'?

Brainstorming

Working in small groups have students create a graffiti chart which considers the question 'What do we meant by this thing called mathematics?' Have each group display their chart and discuss their ideas. Identify common understandings. Ask students to complete *2.1 Activity 1 Student Support Material*, writing their own definition of 'what is meant by this thing called mathematics' and reading the mathematical statements.

Reading

Organise students to work in small groups. Ask each group to choose one of the statements about mathematics found in the *Student Support Material*. Ensure that each group chooses a different statement. Have students discuss the statements and come up with their own explanation of what the statement is saying. Also ask students to decide if they agree or disagree with the statement. *(2.1 Activity 2 Student Support Material)*

Ask each group to share their statement with the rest of the class. Conduct a class discussion about what people have learnt about 'this thing called mathematics'.

Definition

Develop a class definition/understanding of what is meant by 'this thing called mathematics'.

Topic 1- Mathematics and culture

Sample

Direct students to the *Student Support Material Topic 1- Mathematics and Culture.* This provides an example of how one mathematical concept is represented by different cultural groups. Discuss these representations and consider:

- The similarities and differences
- The reasons for variations
- Which one is right? Why?

(2.1 Activity 3 Student Support Material)

Emphasis with students that each culture develops its own way of 'seeking order and pattern in the world' and that each way meets the needs of the particular society.

Group activity

Working in cultural groups ask students to identify some aspects of mathematics that has been developed that represents their own particular worldview (ethnomathematics), and complete *2.1 Activity 4 Student Support Material.*

Students will need to investigate the way their cultural group 'seeks order and pattern in the world of experience, to construct explanations and to intellectualise about this world, and to delight in challenge and in the resolution of problems posed to it by itself'.

Topics which students may investigate include traditional ways of:

- Counting
- Measuring
- Sharing
- Kinship
- Telling the time (during the day, throughout the year)
- Directions
- Trading

Homework Have students prepare a display to share with the class which illustrates the mathematical idea their cultural group has researched.

Display these in the classroom and allow time for students to view and discuss these.

Implication for teaching Consider how the cultural understandings children come to school with compare with the concepts presented in the current primary school curriculum.

- Are they the same or different?
- What are the implications for us as teachers?

(2.1 Activity 5 Student Support Material)

Extension activity

Debate 'Is there a place for traditional mathematical concepts (ethnomathematics) to be taught in schools'?
Debate this issue. Some of the arguments that you may expect students to raise are listed below.

For	Against

- Traditional knowledge is valuable and should be recognized by schools - Our cultural knowledge is fading away and schools have a role to play in teaching this, and ensuring this knowledge is not lost - By recognizing traditional knowledge and valuing its importance students develop pride in their culture - By learning traditional knowledge first, it makes is easier for students to then learn new knowledge without unnecessary confusion	- Cultural ways should be learnt at home and in Elementary School. - It is not the schools responsibility to teach cultural knowledge. - Time to move forward to more 'western ways' rather than retaining traditional ways - There is already too much information to be covered in the curriculum without including traditional knowledge

Class activity Choose one of the mathematical concepts students have researched in *2.1 Activity 4* and consider how this knowledge could be used in a primary classroom context. Ask students to think about:

- How they would teach this concept to children (Who would be involved? What language would you use?)
- If the idea could be used to support the teaching of concepts covered in the primary school curriculum (Why? Why not? Which ideas?)
- What advantages they can see in teaching the concept to children
- What problems they see in teaching the concept

Topic 2 – Language and mathematics

Class activity Teach a short mathematics activity using your Tok Ples (or ask a student). Try teaching an activity which represents your cultural worldview as discussed in Topic 1. Following the activity conduct a class discussion which considers the issues raised by teaching in Tok Ples, and the teaching of ideas from a particular cultural world view.

Issues which you may discuss include:

- If you don't understand the language you have great difficulty in follow the lesson
- When you come from a different cultural background and have a different worldview, it is difficult to make sense of what is being taught
- When you speak the same Tok Ples and share the same worldview then learning becomes easier.

Consider the implications for teaching when we have children in our classrooms coming from different language and cultural backgrounds. Pose the question ' How can we support children from different language and cultural backgrounds learning mathematics in our classrooms?' and ask students to suggest strategies. Keep these ideas to refer to during future lectures

Small group activity

Ask each group to identify a topic in the Lower Primary Mathematics curriculum document and plan a short lesson to be taught in Tok Pisin. Ask one student to teach the lesson to the group. Following the lesson identify the language issues which arose

Issues which you would expect to be discussed would include:

- No words in Tok Pisin for some mathematical concepts such as triangle
- Words used in Tok Pisin to describe an idea have different meaning when used in maths context, which is more precise

As a class share these ideas and discuss the strategies students' adopted/or could adopt to overcome these problems. Add to the ideas suggested in the previous activity.

Homework reading

Refer students to the reading 'The role of language/s in the teaching of mathematics' in the *Student Support Material*, and ask people to read this. Students should identify the important language consideration in the teaching of mathematics and be prepared to discuss these in the next lecture (2.1 A*ctivity 6 and 7 Student Support Material)*

Class discussion

Review the reading with students and identify language considerations in the teaching of mathematics.

Extension activity

Visit to an Elementary School

Organise a visit for students to an Elementary School to observe a mathematics lesson being taught. Ask students to focus on the:

- Language being used
- Content being covered
- Strategies the teacher uses to introduce new language

After the lesson observation ask the class teacher to talk to the students about the language issues faced when teaching mathematics. On return to the classroom share observations from the visit and the implications for teaching.

Topic 3 – Concrete materials

Maths

activities Present a number of mathematics activities for students to complete that would best be solved by the use of concrete materials**. Don't provide students with the concrete materials**. You may wish to set up a number of learning centres around the room and have students work in small groups to rotate around the activities.

Allow students some time to complete the activities. After students have had an opportunity to attempt a solution to the problems, provide them with appropriate concrete materials.

Class discussion Discuss the difficulties experienced when trying to solve problems without the use of concrete materials. Consider how the use of the concrete materials helped.

Homework reading Refer students to the *Student Support Material Topic 3 - Concrete Materials.* Ask students to complete *2.1 Activity 8 and 9* before the next lecture.

Class discussion Discuss the reading and the activity that the students completed. Ask students to comment on the following questions.

- Which age group should use concrete materials when learning mathematics?
- At what stage of the learning process is it best to use concrete materials?

Extension activity *Classroom observation* Organise the students to observe a demonstration lesson which involves children using concrete materials to develop their mathematical understandings. Discuss the lesson with the students and consider:

- The objectives of the lesson
- The concrete materials used
- How the use of concrete materials supported the development of the mathematical ideas
- An evaluation of the lesson and recommendations for improving the lesson

Making a teaching aid Have students develop a teaching aid suitable for use in the mathematics classroom. Students will need to identify the lesson and the mathematical concept which will be taught using the aid.

Topic 4 - Active involvement

Classroom

observation Take students to a school and observe a mathematics lesson. You may wish to break students into small groups and send each group to a different class. During the lesson observation ask the students to focus on what the children and the teacher are doing. Keep a record of what people are doing and if possible the amount of time spent on each activity. For example, children listening to teacher for 10 mins, children talking in small groups for 5 mins, children recording their ideas 10 mins, teacher demonstrating, teacher working with individual children.

Class discussion Following the lesson observation, conduct a class discussion to analysis the findings. Compare observations if students have visited different classes. What conclusions can be drawn about children's involvement in the lessons? What do people see as the advantages of children being actively involved in their learning? Make a list of all the activities which children were involved in.

Homework activity Refer students to the *Student Support Material Topic 4 – Active Involvement.* Ask students to complete *2.1Activity 10.*

Class discussion With the class review the mathematics activities illustrated in *the Student Support Material Topic 4 – Active Involvement*. Discuss the benefits of children participating in these activities when learning mathematics.

Extension activity

Lesson planning Ask students to develop a lesson which demonstrates that they have actively involved children in the learning process. Students should choose a particular grade level and use the curriculum documents to identify a suitable lesson. Peer or micro teacher the lesson if time is available.

Topic 5 - Teaching in context

Class activity Present two sets of activities to students, one activity set in context (relating mathematics to real life experiences, people's needs, interests and goals) and the other not. Both activities should requiring students to use the same mathematical skills to reach a solution

For example,

> **Activity One:** Give students a price list of items from a store and a problem 'You have K5, how many different combinations of items can you buy for your money?'

Activity Two: A set of addition sums using decimals with answers up to 5.00

1.34 + 1.67, .98 + 2.35

Have students solve the problems and as a class consider the following questions:

- What was the purpose of both activities?
- Which activity was more interesting to complete and why?
- What skills were needed to complete both activities?
- Which activity was the most challenging and why?
- Which activity relates more to a real life situation?

Homework reading Refer students to the *Students Support Material Topic 5 – Teaching in Context.* Ask students to complete *2.1 Activity 11,* and *2.1 Activity 12* before the next lecture.

Class discussion Discuss the reading and activity with students. Consider the implications for us as teachers.

Extension activity

Identifying contexts for teaching mathematics Organise students into 6 small groups. Assign a grade level (3 to 8) to each group. Ask each group to review the primary mathematics syllabus document and identify the mathematical topics covered at that level. Ask students to make a list of the contexts they may use to teach the different topics. Consider a range of contexts that will be suitable for children attending schools in all regions of PNG, e.g. Highlands, Coastal.

Conclusion

Review and journal writing Review with students the issues raised in this module and as a class make a list of the issues which influence how children learn mathematics.

Ask students to complete a journal which reflects their understanding of issues which influence how children learn mathematics and the implications for us as teachers.

Module 2.2 – Teaching Mathematics

Module 2.2: Teaching Mathematics is a core module within the unit ***'Teaching and Learning Mathematics'***. During this module students will consider how their beliefs about mathematics influence the approach they adopt when teaching the subject. Strategies for developing children's ability to talk about and record their mathematical thinking will be explored. Co-operative learning, as a strategy for the teaching of mathematics, will be considered together with a range of other strategies appropriate for teaching mathematics in PNG primary schools.

Objectives

By the end of this module students will be able to:

- Discuss their personal philosophy on the teaching of mathematics
- Describe a range of strategies suitable for the teaching of mathematics
- Discuss the advantages and disadvantages of using a co-operative learning approach when teaching mathematics
- Identify a range of different ways children can demonstrate their mathematical thinking
- Construct open ended mathematics questions suitable for use in the primary mathematics classroom

Concepts and skills to be developed

- Teaching skills
- Co-operative learning skills
- Reading and interpreting skills
- Writing skills
- Oral skills
- Mathematical skills
- Questioning skills
- Reasoning and thinking skills

Topics

- Philosophies of mathematics
- Strategies for Teaching Mathematics
- Communicating mathematical ideas (orally and in writing)
- Co-operative learning
- Questioning

Suggested teaching strategies

- Class discussions

- Demonstration lessons
- Peer teaching
- Reading
- Research
- Co-operative group work
- Independent Study

Suggested assessment tasks

- Essay writing: Co-operative learning
  What is your understanding of co-operative learning? Discuss what you see as the advantages and disadvantages of co-operative learning as a strategy for the teaching of mathematics.
- Students plan and teach (microteaching or peer teaching) a lesson incorporating one of the strategies discussed during this module.
- Develop a series of 'good' questions suitable for use in the mathematics classroom. Specify the grade level and the topic being taught.

Resources

- Student Support Material for the unit 'Teaching and Leaning Mathematics'
- Butchers paper and marker pens
- Department of Education Papua New Guinea Mathematics Syllabus Documents for Lower and Upper Primary and the Mathematics Teacher Resource Books and Pupil Books for Grade 3 to 8
- A range of concrete materials such as blocks, shells, bundles of sticks, rulers, scales
- Access to a primary school classroom

References

Booker, George; Bond, Denise; Briggs, Jack; and Davey, Geoff (1997). *Teaching primary mathematics,* Longman

Gould, Peter (1993). *Cooperative Problem Solving in Mathematics, Grade 5 –8,* The Mathematics Association of New South Wales, Inc.

Herrtington, Sparrow, Herrington and Oliver (1997). *Investigating Teaching Strategies in Mathematics classrooms*

Marr, B. and Heme, S. (1991). 'Interaction and Co-operation' in *Breaking the Maths Barrier,* Department of Employment and Training, Canberra, Australia

Mousley, J. and Sullivan, P. (1996). *Learning about Teaching, an interactive tutorial program to facilitate the study of teaching.* AAMT, Deakin University and The Australian Catholic University

A suggested sequence of learning activities

Introduction – Teaching a 'good' mathematics lesson

Sharing experiences Organise students into small groups to share with one another a description of a successful mathematics lesson they have taught or have seen taught (by teachers, other students, and lecturers). After hearing the different descriptions have each group identify the characteristics that made the lessons successful. Report back to the whole class and make a list of the characteristics identified.

Reading Refer students to the *Student Support Material.* Ask students to read the article 'The twelve most important things you can do to be a better maths teacher' and complete *2.2 Activities 1 and 2.* For *2.2 Activity 2* encourage each group to choose a different idea.

Discuss the activities and add to the class list of characteristics of a successful mathematics lesson.

The following characteristics of a 'good' lesson should be considered:

- Teaching in context (relating mathematics to real life experiences, peoples needs, interests and goals)
- Opportunities for children to record their mathematical ideas (charts, journals, pictures, tables, graphs)
- Use of concrete materials
- Opportunities to talk about their learning (group work, reporting to class)
- Well planned
- Building on children's prior learning (moving from the known to the unknown)
- Appropriate level (not too hard, not to easy)
- Challenging
- Students actively involved in the learning.

Topic 1- Philosophies of Mathematics

Reading Refer students to *Topic 1- Philosophies of Mathematics* in the *Student Support Material.* Consider how a philosophy of mathematics influences the way in which we teach mathematics. Ask students to complete *2.2 Activity 3 and 4.*

Draw the following table on the board. Working in small groups have students discuss the reading and complete the table. As a whole class complete the table and discuss.

Philosophy	Belief	Implications for teaching
Formalism		

Logicism		
Intuitionism		

Constructivist Related to an Intuitionist philosophy of teaching mathematics is a constructivist view of mathematics learning. This view has influenced how mathematics is now being taught in many parts of the world. Refer students to the reading 'A constructivist view of mathematics learning' in the *Student Support Material* and complete *2.2 Activity 5*.

Primary Mathematics Syllabus Ask students to review the Primary Mathematics Syllabus for Upper and Lower Primary and read the section 'Teaching Approach'. Identify the philosophical approach to the teaching of mathematics that is implied in the current PNG Primary School Mathematics Curriculum.

Discussion Have students' discuss their own philosophies about the teaching and learning of mathematics in small groups. Ask each group to report back to whole class on their discussions and the implications of these ideas for their teaching. Discuss your philosophy with students.

Extension Activity

Journal writing Ask each student to write their own statement which explains their philosophy of mathematics and how this impacts on their teaching of mathematics. Have students share their writing in small groups. You may wish to collect these journals and select samples for publication in a College newsletter.

Topic 2 – Strategies for teaching Mathematics

Brainstorm With the whole class make a list of the strategies students have used or seen used to teach mathematics. Discuss which of the strategies students see as the most successful and the reasons why they believe this.

Strategies Refer students to the *Student Support Material, Topic 2 Strategies for Teaching Mathematics*. Ask students to complete *2.2 Activity 6.*

Working in small groups have each group of students choose a different strategy and complete the *2.2 Activity 7*. Each group will need time to plan an activity demonstrating the strategy outlined. The activity should be based on a topic taken from the Primary Syllabus. Allow time for students to teach their lesson to the class. Each group will need to identify the strategy they are demonstrating.

Discuss with students the advantages and disadvantages of the different strategies and the situations where each strategy can be best used in the mathematics classroom.

Topic 3 – Communicating Mathematical Ideas

Brainstorming Together with the class make a list of all the ways you can provide opportunities for children to communicate their mathematical ideas. Consider both written (e.g. journal, drawing tables, graphs, charts, diagrams, making up word problems) and oral activities (e.g. talking, explaining, reporting, asking questions, describing, clarifying).

Discussion Discuss with the students the importance of providing opportunities for children to clarify their thinking and further develop their understandings by communicate their mathematical ideas. Also consider how vernacular and English can be used when talking and writing about mathematical concepts.

Refer students to *Topic 3 – Communicating Mathematical ideas* in the *Student Support Material* and complete *2.2 Activity 8.*

Extension activity

School visit Organise a visit to a mathematics classroom and observe a lesson. You may like to divide the class into small groups and each group go to a different class. Ask students to focus on the opportunities children have to communicate their mathematical understandings. Also focus on how mathematical ideas are displayed in the classroom. On return to the class share finding and have students critique the lessons they observed. Consider how the lesson may have been improved to incorporate further opportunities for children to actively participate and communicate their understandings.

Topic 4 – Co-operative learning

Definition Ask students what 'co-operative learning' means to them and together develop a definition. A definition should include ideas such as:

- A group of people working together to reach a common goal
- Students learning from one another
- Each member of the group taking responsibility for and contributing to the achievements of the group goal
- Students of mixed ability learning from one another.

Activity and discussion Present a range of activities which require students to work cooperatively. Sample activities can be found in Annex C.

Following the activities discuss with students their feelings about cooperative learning as a strategy for developing mathematical understandings.

The following questions could be the focus for the discussion:

- What do you see as the advantages of working co-operatively?
- What problems do you see with co-operative learning as a strategy for developing maths understandings? Could these be overcome? How?
- How do you see a co-operative approach to teaching mathematics working in the primary schools?

Homework reading Refer students to the reading in the Student *Support Material 'Interaction and Co-operation'* . Ask students to read the article and complete the activity outlined before the next lecture (*2.2 Activity 9 and 10).*

Discussion Review the article with students and have them share their journals in small groups. Discuss as a class the main ideas raised in the journals.

Extension Activity Primary school syllabus Ask students to work in small groups. Each group will need to choose a grade level and review the mathematics topics to be covered at that level. Students will need to choose a topic and plan an activity which requires the children to work co-operatively. Encourage students to incorporate other ideas suggested previously in the unit into their teaching e.g. use of concrete materials, opportunities for children to communicate their understandings.

Topic 5 – Questioning

Writing questions Present the students with a sample mathematics worksheet (an example can be seen in the *Student Support Material Topic 5 Questioning, Figure 1*). Ask students to work in small groups to list down the questions they could ask when using the worksheet. Have students write the questions on strips of paper and displayed these.

Class Discussion As a class review the questions and consider what makes a 'good mathematics question'. Sort the questions into groups according to people's classification. Discuss the reasons for people's decisions about what makes a good mathematics question.

Reading Refer students to the *Student Support Material, Topic 5 - Questioning* and ask them to complete 2.2 *Activity 11*. After reading the article have students re classify their questions according to whether they are factual, reasoning or open questions. Discuss what the results tell us about the types of questions we ask.

Extension activity

Asking good

questions Have students read the article in the *Student Support Material 'Improving the quality of learning by asking 'good' questions'* and complete the activity which follows *(2.2 Activity 12 & 13)*. Each student will need to choose a topic from the primary School syllabus and develop a series of 'good' questions. Ask people to share their questions with the class. Discuss.

Conclusion

Review and journal writing Review with students the main ideas which have been covered in this module. Ask students to complete a journal outlining what they have learnt about teaching mathematics. Share journals in small groups. Discuss main ideas raised in the journal as a class and the implications for teaching

Annex C: Co-operative Logic Activities

from 'Breaking The Maths Barrier', B. Marr and S. Helme, 1991

Rationale

Co-operative logics are an excellent form of structured group work. They encourage cooperative learning, development of problem solving strategies, and development and reinforcement of mathematical concepts and the language associated with them.

There is a wide range of co-operative logic problems available. The selection we present here is particularly useful for the development of everyday spatial concepts.

It is vital that teachers see these activities in action to give them the confidence and assurance to use them with their own students.

Aims

- To allow participants to experience a range of co-operative logic problem solving activities.
- To develop participants' problem solving strategies, and methods for developing such strategies with their students.
- To increase awareness of the spatial concepts and related language which should be addressed by teachers of literacy and numeracy.
- To provide a focus for the discussion of group work; including its benefits and methods of structuring it for maximum participation.

Materials and equipment

- 1 copy of each problem made up for each small group of 4-6 participants (see preparation)
- 1 set of coloured cubes for each small group: 2 red, 2 yellow, 2 green, 2 blue, 2 orange. These can be centicubes or wooden cubes. Even small MAB cubes coloured with textas will do.

Handouts

1 copy of each of the activities used for each participant.

- *A City Block*
- B *The Flats*
- *C School Fete*
- D-F *Build It 1, 2 and 3*

Time

Allow one hour.

Preparation

Make one copy of each problem for each group of 4-6 participants. Glue each sheet onto card, cut out the clue cards and moveable pieces. Store each problem in an envelope and label it.

Conducting the activity

Divide participants into groups of four. If necessary groups can go up to six in number but this is not desirable for maximum involvement of all participants.

Preliminary Discussion

1. Explain to participants that these co-operative logic activities are for a range of students of varying language ability. We would like them to experience the activities for themselves in order to fully appreciate the problem solving skills, language and concept learning embedded in the problems.
2. Mention that co-operative logics are available at more sophisticated levels of mathematics but in the selection presented here we are catering for the numeracy/literacy and basic mathematics levels. *School Fete* and *Build It* require a more complex reading level and introduce slightly more sophisticated mathematical vocabulary and concepts such as adjacent, edge, and face.
3. Ask participants, whilst doing the problems, to reflect upon the possibilities of using these or adapted versions with their own students - how the activity could act as a springboard, or stimulus for a sequence of learning tasks or lessons in a literacy, numeracy or mathematics class. If time permits at the end of the problem they should discuss their ideas.

Explaining the rules

- Each student should be given at least one clue card. In some problems there are starred cards which can be omitted in the first round and used for checking i.e. *Build It, School Fete* and any EQUALS Publication problems. For others (no starred cards) all the cards are needed and should be distributed, so some group members will receive two cards.
- No one can give their clue cards to another person or lay them down for others to read. They can however, read their card aloud as many times as they, or the group needs, in order to solve the problem.
- Give out one problem at a time to each group. For a group of non-mathematics trained teachers start with the simple City Block and work towards the more difficult ones, such as Build It. It may be a good idea to use the opposite approach if working with a group of maths trained people, i.e. challenge their problem solving skills early with absorbing mathematical problems and then progress to problems more suitable for beginner students.

As the groups work on their problems you should circulate and perform a number of roles:

- Initially clarifying rules and procedure;
- Giving out new problems;
- Discussing the activities as the groups complete them with questions such as:
    - Did you like it?
    - What were its good features?
    - Would it be appropriate for your students?
    - How could it be modified?
    - What other activities could flow from it?
    - How would you have solved it without manipulable (moveable) pieces?
    - Why do you think we provided cut out numbers rather than pencil and paper?

Round up

The main points to be made in discussion are:

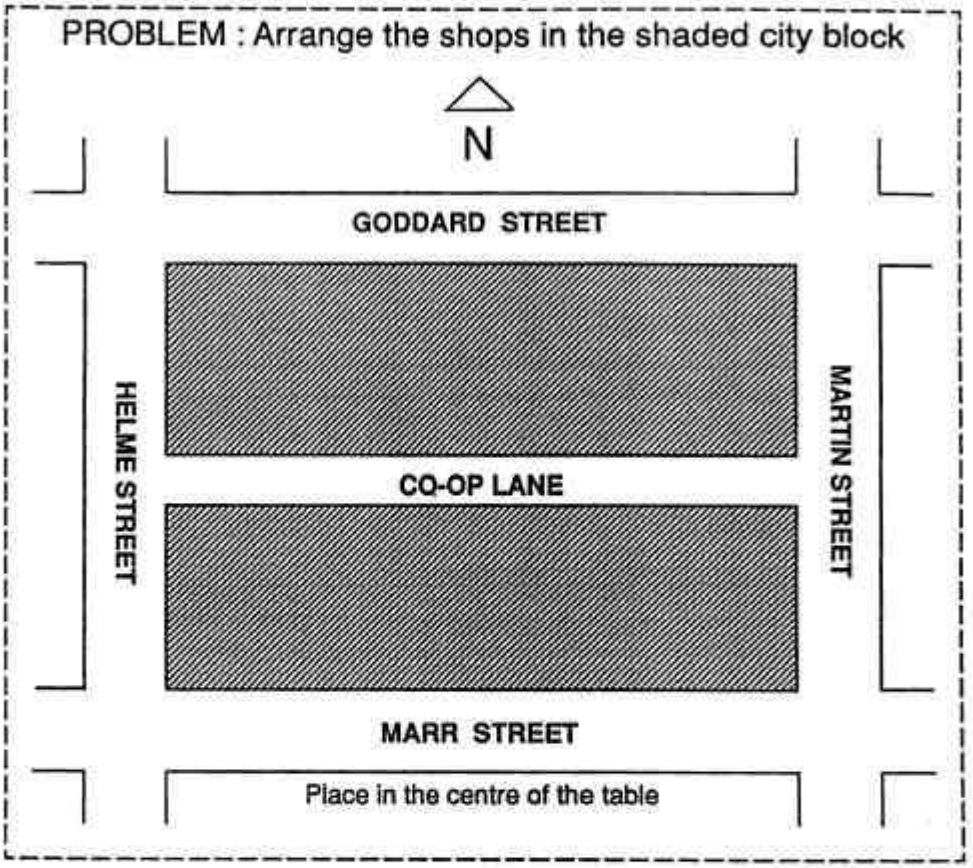
- Having to read their clues aloud ensures that **everyone has a role to play** in the problem solving process. This is why the problem cannot be 'given away' or read aloud by other people. It also encourages people to **listen to each other.**
- Manipulables (movable pieces) encourage **risk taking** and making guesses in the problem solving process.
- Manipulables are also a technique to keep the action in the centre of the group. Pencil and paper tends to allow all the action to move to one end of the table or to be taken over by one or two people. We now discourage paper and pencil for most co-operative logic problems.
- Sometimes it is necessary to discard irrelevant information e.g. *School Fete.*
- This models real life problem solving situations where important information has to be selected and other facts ignored.

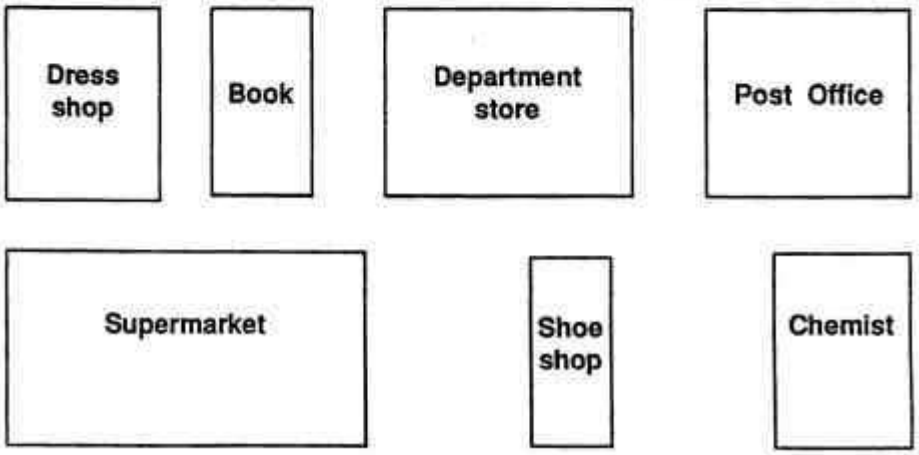
Handout A

Page 1

CITY BLOCK

PROBLEM : Arrange the shops in the shaded city block





1991 Strength in Numbers: Goddard, Marr, Martin

Handout A
Page 2

CLUES - CITY BLOCK

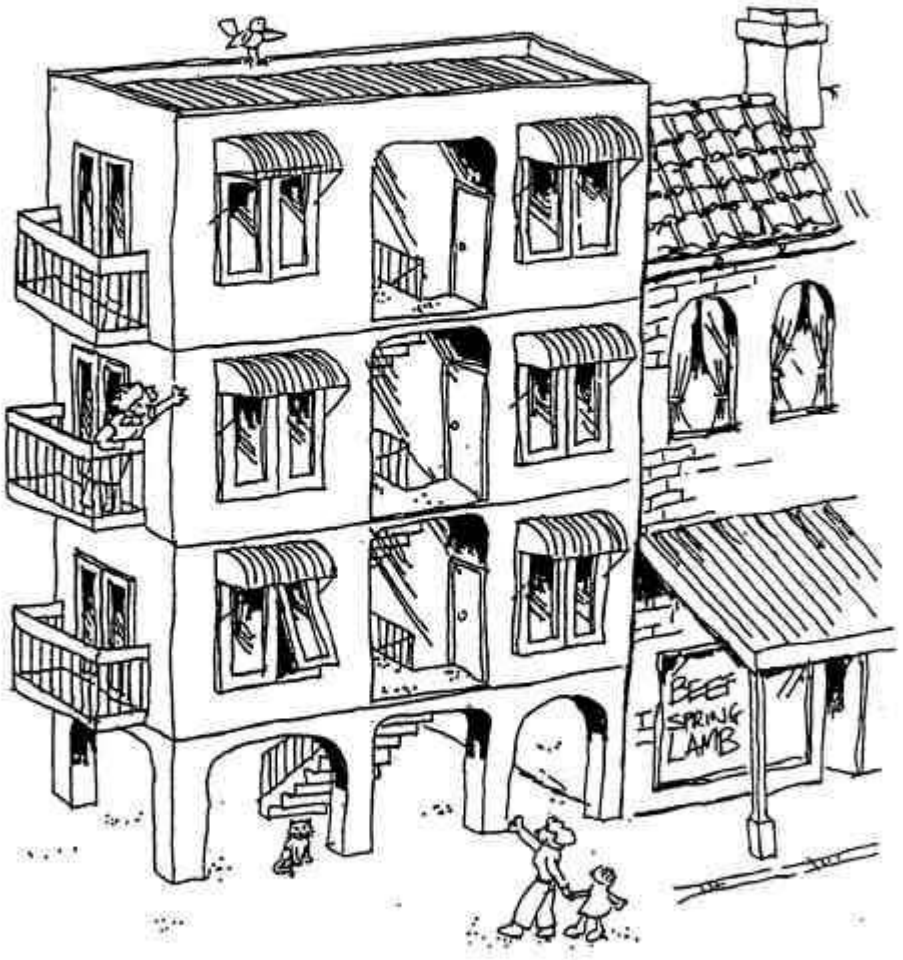
Walking from the supermarket to the chemist, you pass the dress shop.	The post office is south of the chemist, and next to the shoe shop.
The department store and the post office are on corners.	The post office is opposite the dress shop.
The chemist shop is east of the supermarket.	The supermarket is north of the bookshop.

Handout B

Page 1

THE FLATS

(Place this in the centre of the table)



Problem: Who lives in which flat?

1991 Strength in Numbers: Goddard, Marr, Martin

Handout B

Page 2

THE FLATS

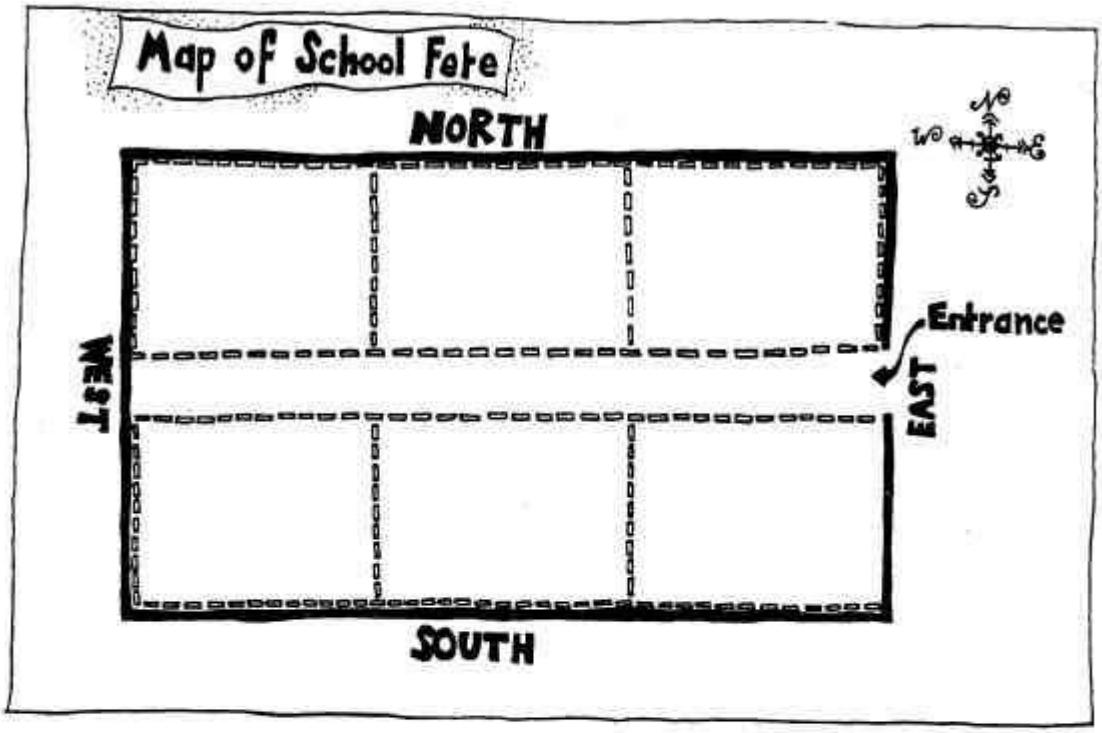
SARTORI	BATES
FISHER	TRAN
JOHANNSEN	WOODS

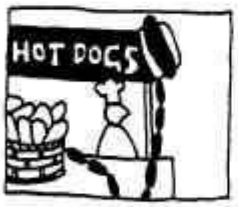
Jo Fisher walks downstairs to feed Maria Sartori's cat when she is away.	The Woods knock on the Tran's floor when their music is too loud.
The Johannsen family hear Mr Wood's feet overhead when he dances.	The Fishers do not live opposite the Johannsen family.
Maria Sartori passes the Bates' flat on the way up to visit the Tran family.	The Fishers grow tomatoes on their balcony in summer.

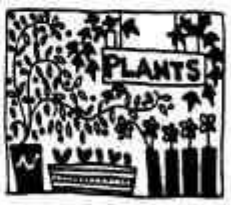
Handout C

Page 1

School Fete







Reproduced from *Mathematics:a New Beginning*, B. Marr and S. Helme 1987

Handout C

Page 2

School Fete

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- The art & craft stall is closer to the entrance than the hot dog stall.
- When Sylvia arrived at the hall, she noticed the clothes stall on her left.

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- The plants are west of the art & craft stall.
- The plant stall is opposite the hot dog stall.
- The plant stall made the most profit.

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- The hot dog stall is next to the clothes stall.
- Joan donated two dozen lamingtons to the cake stall.
- The art and craft stall was not next to the clothes stall.

CLUE CARD: SCHOOL FETE
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- The Annual Fete is held in the school hall every November.
- The clothes are next to the cake stall.
- The lucky wheel is adjacent to the plant stall.

CLUE CARD: SCHOOL FETE*
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at the School Fete?

- When Helen left the hot dog stall she passed the lucky wheel on her way out.
- The cake stall was sold out by lunchtime

CLUE CARD: SCHOOL FETE*
These clues will help solve the problem.

Read them aloud to the group, but do not show them to anyone.

Problem: Where is each stall at theSchool Fete?

- The lucky wheel is north of the clothes stall.
- The first prize on the lucky wheel was a video recorder

*CLUE CARDS marked thus (*) are not necessary for solving the problem. Use these cards if there are more than four players in a group.

Reproduced from *Mathematics a New Beginning*, B. Marr and S. Helme 1987

Handout D

Build It : 1

Build It #1

There are six blocks in all.

One of the blocks is yellow.

Build It #1

The green block shares one face with each of the other five blocks.

Build It #1

The two red blocks do not touch each other.

Build It #1

The two blue blocks do not touch each other.

Build It #1

Each red block shares an edge with the yellow block.

Build It #1

Each blue block shares one edge with each of the red blocks.

Handout E

Build It: 2

Build It #2

There are six blocks in all, in a tower six blocks high.

There is a yellow block on top.

Build It #2

The red block is above the green block.

Build It #2

One of the yellows is above the green block; the other is below it.

Build It #2

Each of the blue blocks shares a face with the green block.

Build It #2

No two blocks of the same color touch each other.

Build It #2

There are two yellows, two blues, one green, and one red in the set of blocks.

Handout F

Build It: 3

Build It #3

There is a red block directly below a yellow block.

There is a green block on the bottom level.

Build It #3

There is a red block directly on top of a yellow block.

The highest block is on the third level.

Build It #3

There are six blocks in all.

An orange block shares a face with a green block and two others.

Build It #3

A blue block shares a face with a yellow block.

There is a red block on the bottom level.

Build It #3

A blue block touches red and green blocks only along edges.

There are three blocks on the bottom level.

Build It #3

A yellow block touches an orange block only along an edge.

Module 2.3 – Syllabus Studies

Module 2.3 Syllabus Studies is a core module in the unit ***'Teaching and Learning Mathematics'***. During this module students will examine the 'Mathematics Syllabus for

Lower Primary, Grade 3-5' and the 'Mathematics Syllabus for Upper Primary Grades 68'. Students will further develop their skills in programming, planning, teaching, assessing children's learning and evaluating mathematics activities, which are gender inclusive and give consideration to catering for children:

- Bridging to English
- With special needs • In multigrade classes.

Objectives

By the end of this module students will be able to:

- Interpret the primary school mathematics syllabus
- Use the primary school mathematics syllabus and Teacher's Resource Books to develop a mathematics program
- Identify a range of strategies which are gender inclusive and suitable for teaching children who are: - bridging to English, - have special needs, - in a multigrade class.
- Plan, teach and evaluate a series of mathematics activities
- Identify appropriate assessment procedures suitable for use in the primary school

Concepts and skills to be developed

- Programming skills
- Teaching skills
- Assessment skills
- Evaluative skills
- Interpretation skills

Topics

- The Primary School Curriculum
- Planning to Teaching Mathematics
- Gender Inclusive Education
- Bridging from Vernacular to English
- Children with Special Needs
- Teaching in a Multigrade Classroom
- Assessment

Suggested teaching strategies

- Class discussion
- Small group work
- Lesson observations
- Peer teaching
- Micro teaching

- Oral presentations
- Independent study

Suggested assessment tasks

- Planning of a unit of work for a particular grade level
- Plan, teach and evaluate a maths lesson which is gender inclusive and caters for
  - children in a multigrade class or - children bridging to English or
  - a child with a special need
- Outline 4 different methods of assessing children's mathematical learning and give an example of a situation when you would use each method.

Resources

- Student Support Material for the unit 'Teaching and Leaning Mathematics'
- Butchers paper
- Marker pens
- Department of Education Papua New Guinea Mathematics Syllabus Documents for Lower and Upper Primary
- Mathematics Teacher Resource Books and Pupil Books for Grade 3 to 8
- A range of concrete materials such as blocks, shells, bundles of sticks, rulers, scales
- Access to a primary school classroom

References

Booker, George; Bond, Denise; Briggs, Jack; and Davey, Geoff (1997). *Teaching primary mathematics,* Longman

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*,

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Department of Education Papua New Guinea (1999). *Mathematics Teacher's Resource Book Grade 4,* Oxford University Press

Ministry of Education (Schools Division) Victoria (1988). *The Mathematics Framework P-10,* Department of Education Victoria, Australia

A suggested sequence of learning activities

Introduction – Mathematics in the Elementary Schools

Elementary School Mathematics If possible visit an elementary school with the students and spend some

time observing the children learning mathematics. Allow an opportunity for the Elementary teachers to discuss with the students what children are learning in mathematics and the approach used to teach mathematics. Allow time for students to familiarise themselves with the Elementary Mathematics curriculum.

If it is not possible to visit an Elementary school you could invite an Elementary teacher to be a guest speaker. Ask them to bring along examples of the children's work, along with the Elementary Curriculum materials used for teaching mathematics. The Elementary teacher could explain to the students the approach used to teach mathematics and the mathematical knowledge children are learning during their time in Elementary School.

Journal Writing

Have students complete a journal sharing their ideas about what they have found out about mathematics in the Elementary school. Share journals and highlight the fact that children will enter primary school with a strong foundation for further learning.

Topic 1- Syllabus documents

Mathematics Resources

Prepare a display of mathematics resource materials used in Primary Schools to teach mathematics. This should include the Community School Mathematics books, the Lower and Upper Primary Mathematics Syllabus (1999), and the new Mathematics Teacher's Resource and Pupil books (e.g. the Grade 4 material) as they become available. Also include any other useful mathematics resources which you have access to. Allow students time to look through the resource material.

Discussion

Review the resources with the class and discuss how classroom teachers use these. In particular highlight resources which are the most current PNG Department of Education publications for use in primary schools.

Syllabus Review

Organise students into small groups to examine the primary mathematics curriculum syllabus documents (Lower and Upper Primary). Identify the different components in each document and the information contained in the various sections. Consider

- The understandings which inform the document (rationale, assumptions)
- The teaching approaches suggested (including multigrade, integration, context)
- The goals, aims and objectives
- The scope and sequence
- Detailed objectives
- Assessment approaches

Have students make notes on the above areas, and follow up with a class discussion. Emphasis needs to be place on how these documents are used by teachers to develop their mathematics program.

Teacher and pupil books Discuss with the students the relationship between the syllabus documents and the Teacher's Resource Books and Pupil Books. Refer Students to the *Student Support Material Topic 1- Syllabus Documents* and complete *2.3 Activity 1*.

Using the resources Provide opportunities for students to locate information in the documents and develop an understanding of how the different resources relate to one another.

For example, if you were teaching Grade 4 measurement and the topic 'area' what objectives would you need to address? What units in the teacher's resource book cover these objectives? What are some of the new words you would focus on when teaching this unit? What activities are there in the Pupil book to help you teach this topic?

Extension activity Guest speaker Invite a practicing teacher to speak to the students about how they use the syllabus documents to plan mathematical activities. Allow opportunities for discussion and questions.

Topic 2 – Planning to teach mathematics

Teaching maths in context Refer students to *Topic 2 – Planning to Teach Mathematics* in the *Student Support Material*. Review the Planning Model with the students and the importance of teaching mathematics in context and making mathematics purposeful to children. Allow students an opportunity to complete *2.3 Activity 2*. Talk through each step in the model and answer any questions which students may have.

Joint Construction Together as a class plan a unit of work using the planning model. Work through each step and model the process with students.

Small group activity Organise students into small groups and ask them to plan a unit of work for a particular grade level. Encourage different groups to work on different grade levels. Students are to use the curriculum documents and the planning model.

Sharing Allow an opportunity for students to share their work with the remainder of the class. Display programs. You may wish to select good examples to collate and distribution to students so they can be used when students are teaching.

Discussion Discuss the planning model with the students and identify what people see as strengthens and problems associated with using this approach to planning mathematics. Consider how the problems identified can be overcome.

Extension activity Micro teaching Ask each student to teach one of the activities developed using the planning model to a small group of children. Students will need to evaluate their teaching with particular focus on

- the context they used
- the language they used
- the children's learning

Discuss lessons together as a whole class.

Topic 3 – Gender Awareness

Review Discuss with students what their understandings are about the terms such as 'gender', 'gender equity' and 'equity in education'.

Ask students to indicate if they studied mathematics in Year 12 and see if there is a difference between the number of male and female students. When mathematics is not compulsory girls generally are less likely to choose to study mathematics. Ask students to consider why this often is the case. Discuss why it may be a disadvantage not to study mathematics at an Upper Secondary level

e.g. - educational and occupational opportunities are reduced
- may feel less confident as citizens in a technological society

Refer students to *Topic 3 – Gender Awareness* in the *Student Support Material.* Allow students an opportunity to complete *2.3 Activity 3*

'***A National Statement on Girls and Mathematics***' (Australian Association of Mathematics Teachers, July 1990) is a useful resource for this activity

Topic 4 – Bridging from vernacular to English

Review prior knowledge Ask students to recall issues covered earlier in the unit which focused on language and mathematics and communicating mathematical ideas. Discuss the understanding students have developed.

Reading — Refer students to the *Student Support Material, Topic 4 - The Language of Mathematics.* You may like to ask students to read the article for homework prior to teaching this lesson. Allow time for students to discuss the article and complete *2.3 Activity 4.*

Class Discussion — Discuss the ideas contained in the reading and the implications for teaching.

Activity — Provide students with sample mathematics activity which highlights new language which needs to be taught to children e.g. samples are provided in the Grade 4 Mathematics Teachers Resource Book (1999). Working in small groups, ask students to identify the strategies they would use to introduce the new language to children. Share strategies and discuss their effectiveness.

Do, talk record — Model a mathematics lesson to the students which demonstrates the 'Do, Talk, Record' strategy. Discuss the approach with students and have them complete *2.3 Activity 5 in the Student Support Material.*

Extension activity

School visit — Organise a visit to a Grade Three class so students can observe a mathematics lesson. Take note of the language the teacher is using and how new words are being introduced to children. If the teacher is available to speak with students ask the teacher to talk about the strategies they have used to support children bridging to English when teaching mathematics.

Micro teaching — Ask students to teach a mathematics lesson to a small group of Grade Three children. This could be the lesson planned in *2.3 Activity 5.* When the students are teaching the lesson have them focus on:

- the language they used to teach the lesson
- how any new language was introduced to the children.
- The relationship between doing, talking and recording

After students have taught the lesson ask them to share their experiences and to discuss the language issues which arose.

Topic 5 – Catering for Children with Special Needs

Share experiences — Discuss with students what is meant when we refer to children as having 'special needs'. Ask students to share experiences they have had with children with special needs (in the community or in the classroom).
Consider how parents/teachers cater for children with special needs. Share the following statement with the students and discuss.

> *The fundamental principle of the inclusive school is that all children should learn together wherever possible, regardless of any differences or difficulties they may have. Inclusive schools must recognise and respond to the diverse needs of their students, accommodating both different styles and rates of*

learning and ensuring quality education to all through appropriate curriculum, organisational arrangements, teaching strategies, resource use and partnerships with their communities. There should be a continuum of support and services to match the continuum of special needs encountered within the school. (UNESCO Conference 1994)

Highlight to students that as teachers we have a responsibility to cater for all children in our classes and the need for us to develop strategies to support children with special needs.

Working with students' make a list of the types of 'special needs' children attending primary schools are likely to have e.g. hearing, visual, physical, intellectual.

Reading Refer students to the *Student Support Material Topic 5 – Catering for Children with Special Needs.* Allow students an opportunity to read the article. You may like to ask students to read the article for homework prior to teaching this lesson. Ask students to discuss the lesson outlined and the strategies adopted to cater for the children with the specified special needs. (*2.3 Activity 6 Student Support Material*)

Class discussion Identify the strategies suggested in the reading and the importance of focusing on what the child can do, rather than what they can't do. Ask students to consider how they feel these strategies would work in a primary school situation. Identify any other strategies which students believe would support children with special needs.

Extension activity

Lesson planning Working in small groups ask students to plan a mathematics lesson which caters for a child with a special need. Encourage each group choose a different special need. Share lessons and identify the strategies people used to support children to learn.

Topic 6 - Teaching in a Multigrade Classroom

Definition Ask students to explain their understanding of what is meant by the term 'a multigrade class'. Work with students to develop a shared understanding of the term. If students have visited a multigrade classroom or were in a multigrade class when attending school, ask them to share their experiences

Reading Refer students to the *Student Support Material Topic 6 – Multigrade*. You may like to ask students to read this section for homework prior to teaching this lesson. Allow students time to discuss the article and complete *2.3 Activity 7* in the *Student Support Material.*

Lesson presentation Plan and teach a lesson to the class which demonstrates teaching in a multigrade class. For example you may

- set up a number of different learning centres which cater for different ability groups

- plan a co-operative learning activity which allows students with different abilities to support one another
- divide the students into different groups and plan separate activities for the groups demonstrating small group organisation or the peeling off strategy

Class Discussion Discuss with the students the lesson you modelled and identify the strategies used and their effectiveness. Ask students to identify the advantages and disadvantages they see in operating a multigrade class.

Extension activity

School visit Visit a multigrade class or ask a teacher who is working in a multigrade class to talk to students. Identify the strategies teachers are using to cater for children in a multigrade classroom.

Lesson Planning Ask students to use the curriculum documents to plan a mathematics lesson to cater for children in a multigrade class. Share these as a class and discuss strategies.

Topic 7 - Assessment

Sharing ideas Provide students with strips of paper and ask them to write down what they see as the main purposes of assessment. Encourage each student to write down more than one idea. Display students' comments and group them according to similar ideas. As a class make a list of the main purposes of assessment identified.

Brainstorming Together as a class make a list of all the different ways you are able to assess a child's learning. Ask students to recall their experiences when on block teaching or school experience. These should include both formal and informal assessment activities e.g. observations, listening to children talking, tests, homework.

Reading Refer students to the *Student Support Material Topic 7 – Assessment Issues* and allow students time to read the article. Discuss the main ideas raised in the article and add any new ideas to the lists developed in the previous two activities. *(2.3 Activity 8 Student Support Material)*

Assessment activities Review the Primary School Syllabus document. Working in small groups have students choose a particular grade level and topic. From the suggested objectives and activities outlined ask students to think about how they might go about assessing children. Ask each group to outline a number of different ways they might collect information which demonstrates a child's understanding of a particular objective. Share these as a class and discuss. Consider how this information would be recorded.

Extension activity

Guest Speaker

Invite a practicing classroom teacher to come to the class and talk to students about the procedures they use to assess children. Ask the teacher to bring along examples of their assessment records and to discuss with students how this information is collected, recorded and used.

Conclusion

Review

Ask students to recall the main ideas they have learnt from this module. Ask each student to write down one new idea which they have learnt from the module which will help them in their teaching of mathematics. Also ask each student to write down one topic which was covered during the module that they wish to learn more about. Display these ideas and discuss. Address any questions students may have about the material covered during the module.

Unit Glossary

concrete materials These are materials which students can physically hold onto and move round. Examples of concrete materials include such things as blocks, sticks, shells, money and counters. Concrete materials are very useful when introducing new concepts and assist students in solving a range of problems. Concrete materials can be used with students of all ages e.g. from Elementary to Tertiary.
Concrete materials are also referred to as manipulative materials.

co-operative learning Co-operative learning is a strategy which involves a group of students working together to reach a common goal.

Ethno-Mathematics Ethnomathematics refers to the mathematical ideas belonging to a particular 'culture' or 'race'. 'Ethno' means 'race' or 'nation'. PNG mathematical counting systems are an example of ethnomathematics.

Formalism Formalism is a philosophical approach to teaching mathematics. People who have a formalist approach to teaching mathematics believe that mathematics is a set of characteristics (e.g. 1, 2, 3, 4, +, =) which are joined together in an organised way to form formulas. These formulas have certain rules and to learn mathematics you need to learn all these rules and formulas.

Intuitionism Intuitionism is a philosophical approach to teaching mathematics. People who are intuitionist believe that mathematics is made up of a series of symbols which have meaning and represent ideas (e.g. 1 which represents one object, + which means add a number of objects together, = which is the symbol which means equals). To learn mathematics intuitionist believe you need to develop your own understanding of what the mathematics symbols mean through constructing your own knowledge. This means using everyday language to talk about mathematical ideas, using concrete materials to develop mathematical concepts and then learning about how to use the symbols of mathematics to express these ideas.

Logicism Logicism is a philosophical approach to teaching mathematics. People who are logicists believe that learning mathematics involves logic and reasoning. To learn mathematics logicists believe you need to learn the logic and reasoning behind the formulas.

real life experiences This refers to when you have first hand experience of something. For example rather than hearing or reading about what life it like in a particular place, you actually go there yourself and experience the place.

realistic experiences this refers to a situation which you set up to imitate as much as possible a real situation. For example in a classroom you may set up a class shop which is similar to a trade store.

teaching in context When you relate your teaching to real life situations and make connections between what you are teaching in the classroom and what happens in the real world outside the classroom, this is referred to as teaching in context.

techno-mathematics This refers to the mathematical understandings which are shared by people around the world, and are reflected in technological developments. This is the type of mathematics which we teach through the syllabus documents in our schools and represents some ways in which technological cultures view the world.